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Fitting the term structure of interest rates in illiquid market: Taiwan experience

Abstract

This paper aims to compare the fitting performance of term structure estimation for Taiwan Government Bonds market, which is considered as an illiquid bond market with a low trading volume, based on the Nelson and Siegel, Extended Nelson-Siegel Model and Nelson-Siegel-Svensson Model (see Nelson and Sigel, 1987; Bliss, 1996; Svensson, 1994). The empirical results indicate that the fitting performance in accuracy for Nelson-Siegel-Svensson Model is better than that of Extended Nelson-Siegel Model, and the Extended Nelson-Siegel Model is better than that of Nelson-Siegel Model. It means that adding more parameters will obtain a better capability in describing the shape of the term structure. Also, compared with the case of which the liquidity constraint is not taken into consideration, these three models will have a better fitting performance if the liquidity constraint is considered.

Keywords: term structure of interest rates, Nelson-Siegel Model, liquidity constraint.
JEL Classification: G12, G15.

Introduction

The relationship between the yields of default-free zero coupon bonds and their length to maturity is defined as the term structure of interest rates and is shown pictorially in the yield curve. This relation can be used for risk management and has an important role in pricing fixed-income securities and interest rate derivatives, as well as other financial assets. Because of its numerous uses, an accurate estimate of the term structure has constituted a major question in the empirical literature in economics and finance.

Many alternative estimation methods for yield curves appeared in the literature over the years. Generally speaking, there are two distinct approaches to estimate the term structure of interest rates: the equilibrium models and the statistical techniques. The first approach is formalized by defining state variables characterizing the state of the economy (relevant to the determination of the term structure) which are driven by these random processes and are related in some way to the prices of bonds. It then uses no-arbitrage arguments to infer the dynamics of the term structure. Examples of this approach include Vasicek (1977), Dothan (1978), Brennan and Schwartz (1979), Cox Ingersoll and Ross (CIR, 1985) and Duffie and Kan (1996). Unfortunately, in terms of the expedient assumptions about the nature of the random process driving the interest rates, the yield curves derived by those models have a specific functional form dependent only on a few parameters, and usually the observed yield curves exhibit more varied shapes than those justified by the equilibrium models.

In contrast to the equilibrium models, the statistical techniques focusing on obtaining a continuing yield curve from cross-sectional coupon bond data based on curve fitting techniques are able to describe a richer variety of yield patterns in reality. The resulting term structure estimated from the statistical techniques can be directly put into interest rate models, such as the Ho and Lee (1986), the Heath et al. (1992) and Hull and White (1990) models, for pricing interest rate contingent claims. Since a coupon bond can be considered as a portfolio of discount bonds with maturities dates consistent with the coupon dates, the discount bond prices thus can be extracted from actual coupon bond prices by statistical techniques[1]. These methods can be broadly divided into two categories: the splines and the parsimonious function forms (see Alper et al., 2004). Spline-based models were first proposed by McCulloch (1971, 1975) who used various piecewise polynomial splines to estimate the discount function. He found that the discount function could be fitted very well by cubic or higher order splines and the estimated forward rates are a smooth function. Schaefer (1981) uses a set of Bernstein polynomials to appropriate the term structure. Even though polynomial splines constitute a very flexible family of curves, they do not constrain the discount function to be non-increasing. Vasicek and Fong (1982) use a third-order exponential spline to estimate the discount function and claim such models are superior to polynomial splines models. However, Shea (1985) points out that there is no evidence to support that exponential splines

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[1] Once the discount function, $P(t)$, is defined, the spot interest rate (the pure discount bond yield) can be computed by:

$$R(t) = \frac{-\ln P(t)}{t}.$$

produce more stable estimates of the term structure than polynomial splines. Later, Mastronikola (1991) develops a more complex cubic splines and concludes the model's fit is superior to that of its predecessor. In terms of previous considerations, an optimal approximation for a discount function can be found as a linear combination of elements of the basis which would be constructed by the B-spline function. In practice, the B-spline function has been successfully used by Steeley (1991), the UK Gilted-edged term structure. Lin and Paxson (1993) also apply this methodology to estimate the German government bond term structure and conclude that the B-spline function can appropriately approximate the discount function and result in reliable and smooth spot and forward rate curves. Deacan and Derry (1994) conclude that the consensus view in the literature appears to have a preference for the use of B-splines after examining various techniques used for the term structure estimation. Moreover, Jarrow, David and Yu (2004) use a semi-parametric penalized spline model to estimate the term structure of corporate bond, Krivobokova et al. (2006) use the penalized splines to analyze the term structure of interest rates extracted from US Treasury STRIPS data.

Parsimonious models, on the other hand, specify a parsimonious parameterizations of the discount function, spot rate or the implied forward rate. Chambers et al. (1984) consider an exponential polynomial to model the discount function. Nelson and Siegel (1987) choose an exponential function with only four unknown parameters for modeling the forward rate of U.S. Treasury bills, unlike the spline class that models the discount function. By considering the three components that make up this function, Nelson and Siegel (1987) illustrate that it can be used to generate forward rates curves of a variety of shapes and analytically solve for the spot rate. Moreover, the advantage of Nelson-Siegel model is that three parameters may be interpreted as latent level, slope and curvatures factors. Dield et al. (2005), Diebold and Li (2006), Diebold et al. (2006), Modena (2008), and Tam and Yu (2008) have employed the Nelson-Siegel interpolant to examine bond pricing with a dynamic latent factor approach.

The Extended Nelson-Siegel method, defined by Bliss (1996), introduces a new estimation method to fit a modified version of the appropriating function with five parameters developed by Nelson and Siegel (1987). Bliss suggests that a five-parameter specification can produce better results for fitting the terms structure with longer maturities. Svensson (1994) increases the flexibility of the original Nelson and Siegel model by adding two extra parameters (hereafter Nelson-Siegel-Svensson model) and allows for a second "hump" in the forward rate curve.

The objective in empirical estimation of the term structure is to fit the data sufficiently well and, at the same time, obtain a sufficiently smooth and continuous function as the term structure of interest rates (Lin, 2002). There has been considerable effort expanded in comparing the fitting performance of alternative methods of yield curve estimation. For example, Bliss (1996) compares five diverse methods (the Unsmoothed Fama-Bliss (1987), McCulloch cubic spline, Fisher-Nychka-Zervos cubic spline (1995), Extended Nelson-Siegel, and the smoothed Fama-Bliss) for estimating the term structure and finds that the Unsmoothed Fama-Bliss does best overall. Anderson et al. (1996) compare four methods of yield curve estimation (Mastronikola, McCulloch, Nelson and Siegel, and the Nelson-Siegel-Svensson model) and tend to favor the Mastronikola model. Jeffrey et al. (2000) conclude that a nonparametric kernel smoothing procedure to fit the discount function developed by Linton et al. (2000) overall performs notably better than the highly flexible McCulloch (1975) cubic spline and Fama-Bliss (1987) bootstrap methods. Yeh and Lin (2003) apply two equilibrium models: the Vasicek and the CIR model, and one statistical technique: the B-spline approximation function, as the discount function to extract the term structure from market coupon bond prices. They find that, although the equilibrium model can contain economic information and is able to explain the term structure dynamics, the statistical technique can fit the term structure better than the equilibrium model. Ioannides (2003) examines different methods of estimating the term structure rates on daily UK Treasury bills and gilt data. The Nelson-Siegel-Svensson functions, McCulloch's Cubic spline, the linear, exponential and integrated exponential B-spline and the VRP method, a total of seven methods are used to test their fitting performance. Ioannides suggests that the parsimonious specifications and VRP method perform better than the linear spline counterparts form in-sample and out-of-sample analysis of residuals.

Clearly, the empirical results reviewed above indicate that it is impossible to identify one method being definitively superior to all others. Since each methodology has its strengths and weaknesses, the choice of which model should be used depends on one's subjective preferences. This paper aims to estimate and analyze the Taiwan government bond (TGB) term structure of interest rates based on the parsimonious functions specification, i.e. the four parameters Nelson-Siegel model, the five parameters Extended Nelson-Siegel method, and the six parameters Nelson-Siegel-Svensson model. The reason why we choose the Nelson-Siegel families is these models have substantial flexibility required to match the changing shape of the yield curve, yet they only use few parameters. As noted by Diebond and Li (2006), it can be used to predict the future level, slope, and curvature factors for bond portfolio investment purposes. To the best of our knowledge, no one has investigated the over parameters problem for these models, i.e., is it beneficial to the term structure fitting performance when adding one or two extra parameters compared to the original Nelson and Siegel model. In addition, previous literature indicates that although there are lots of curve fitting models that have been successfully applied to developed bond markets, particularly in US government bonds and treasury bills market, however, comparatively little attention has been paid to emerging markets (for example, Alper et al., 2004; Dutta et al., 2005; Cortazar et al., 2007). The developed bond markets are generally well established and comprised of relatively liquid securities with short and long maturities. However, in the developing economies with sparse bond market price data, a substantial portion of the secondary market trading is concentrated in a handful of bonds that the market perceives liquid, thus it is not meaningful to estimate the term structure based on a small number of liquid securities. Subramanian (2001) is the pioneer in positing a model for the yield curve estimation based on a liquidity-weighted objective functions to ensure that liquid bonds in the market are priced with smaller errors than the illiquid bonds.

Compared to other developed countries' bond markets, Taiwan bonds and bills market has a noticeably smaller trading volume and is not liquid. In financing national development projects, the government has begun to issue bonds in large volume since 1991. And since then, the bond market has gathered more and more volume in both the primary and secondary markets. In 2007, the trading volume of bond's secondary market reached NT$ 194 trillion[1], as compared to the stock market's NT$ 26.1 trillion; the scale of bond trading is about 7.4 times that of the stock market, showing that the Taiwan bond market has truly expanded. Figure 1 shows the movements of 10-year Treasury bond yield and 10-day Commercial paper interest rate.

Recently, to accelerate the pace of liberalization and internationalization, the authorities have greatly eased the regulations and thus improved the trade efficiency in the secondary market. In order to attract more foreign interest and further develop Taiwan as an Asian-Pacific regional financial center, the Taiwan Futures Exchange (TAIFEX) launched its operation in July of 1998 and introduced a local 10-year Government Bond Futures for hedging and speculation purposes. After the strenuous efforts of several years, the Taiwan financial market has been placed on the top of the list of fast-growing markets in the world.

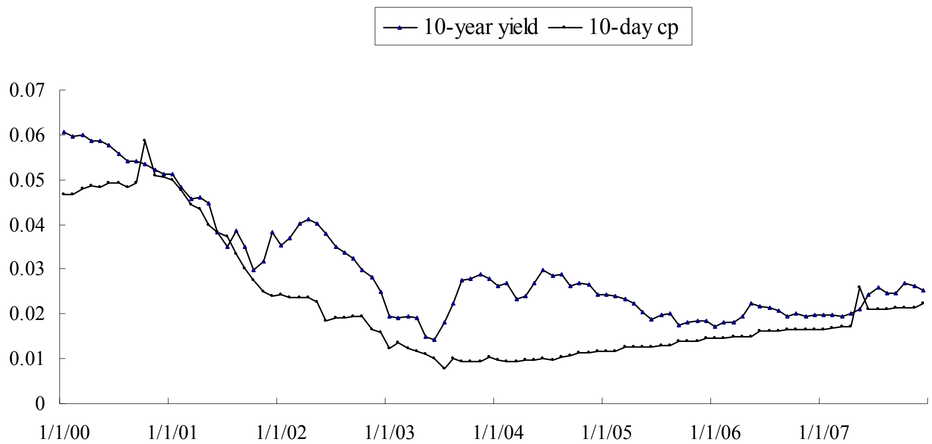

Fig. 1. Time path of the 10-year Treasury bond yield and 10-day Commercial paper interest rate

This study is the first of its kind to use a family of Nelson-Siegel yield curve models for estimating the term structure in an emerging market economy, Taiwan. Meanwhile, we will provide the empirical results for both fitting performance with and without considering liquidity constraint. Following this brief introduction, Section 1 is the empirical methodology. Section 2 presents the empirical results. And the concluding remarks are given in the last section.

1. Empirical methodology

Assume that there are n default-free coupon bonds in the sample to estimate the term structure of interest rates. Because the value of the coupon bond i, B_i is simply the present value of the stream of future cash flows it provides. That is,

$$B_i = \sum_{m=1}^{T} \frac{CF_{im}}{[1 + R(m)]^m}, \qquad 1 \le i \le n, \tag{1}$$

where CF_{im} is the cash flow paid by bond i occurring at time $m = 1,\ldots,T$ (the maturity of the bond),

[1] The average exchange rate is US$1= NT$ 32.842 in this year.

and $R(m)$ is usually referred to as the spot interest rate for maturity *m* years.

1.1. Nelson-Siegel Model. The Nelson-Siegel Model chooses a function form of the forward rate curve that allows it to take a number of shapes. The instantaneous forward rate at maturity *m* is given by the solution to a second-order differential equation with real and equal roots. The function form they suggest is:

$$f(m) = \beta_0 + \beta_1 \exp(\frac{-m}{\tau}) + \beta_2[(\frac{m}{\tau})\exp(\frac{-m}{\tau})]. \quad (2)$$

The first term, β_0, represents the long-term value of the interest rate. The second and third terms, β_1 and β_2, indicate the slope and curvature parameter. The time constant τ is the scale parameter that measures the rate at which the short-term and medium-term components decay to zero. For example, small values of τ result in rapid decay in the predictor variables and therefore will be suitable for curvature at low maturities. Corresponding, large values of τ produce slow decay in the predictor variables and will be suitable for curvature over longer maturities (Christofi, 1998). Following McCulloch's (1971) definition of the yield as an average of the forward rate, the spot interest rate for maturity *m* can be derived by integrating Eq. (2) from zero to *m* and dividing by *m*. The resulting function can be expressed as follows:

$$R(m) = \beta_0 + \beta_1\left(\frac{\tau}{m}\right)\left[1 - exp\left(\frac{-m}{\tau}\right)\right] + \beta_2\left(\frac{\tau}{m}\right)\left[1 - exp\left(\frac{-m}{\tau}\right)\left(\frac{m}{\tau}+1\right)\right]. \quad (3)$$

Unfortunately, the true underlying term structure is unobservable in TGB market. However, in an efficient market, a correctly specified term structure estimation model would exactly explain the observed bond prices for all maturities. On the bond markets we could observe deviations between the market price (quoted price plus accrued interest) and the theoretical model price given an estimated term structure of interest rates as follows:

$$P_i = B_i + \varepsilon_i, \qquad 1 \le i \le n \quad (4)$$

where P_i denotes the market price of the coupon bond *i*, ε_i then is the pricing error of bond *i*[1].

Combining Eqs. (1), (3) and (4) results in Eq. (5):

$$P_i = \sum_{m=1}^{T} \frac{CF_{im}}{\left\{1+\beta_0+\beta_1\left(\frac{\tau}{m}\right)\left[1-exp\left(\frac{-m}{\tau}\right)\right]+\beta_2\left(\frac{\tau}{m}\right)\left[1-exp\left(\frac{-m}{\tau}\right)\left(\frac{m}{\tau}+1\right)\right]\right\}^m} + \varepsilon_i. \quad (5)$$

Thus, we can estimate the above four parameters, $\varphi \equiv \{\beta_0, \beta_1, \beta_2, \tau\}$, embedded in Nelson-Siegel model using the following nonlinear, constrained optimization estimation procedure based on the modified Gauss-Newton numerical method (see Hartley, 1961):

$$min\sum_{i=1}^{n}(P_i - B_i)^2 = min\sum_{i=1}^{n}\varepsilon_i^2. \quad (6)$$

1.2. Extended Nelson-Siegel Model. The Extended Nelson-Siegel Model sets the instantaneous forward rate at maturity *m* given by the solution to a second-order differential equation with unequal roots as follows:

$$f(m) = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \exp\left(\frac{-m}{\tau_2}\right), \quad (7)$$

where the unknown parameters β_0, β_1, and β_2 have the same economic interpretation as the Nelson-Siegel model, the parameters τ_1 and τ_2 determine the speed of convergence for β_1 and β_2.

The spot rate can be derived by integrating the forward rate and is given by

$$R(m) = \beta_0 + \beta_1\left(\frac{\tau_1}{m}\right)\left[1 - exp\left(\frac{-m}{\tau_1}\right)\right] + \beta_2\left(\frac{\tau_2}{m}\right)\left[1 - exp\left(\frac{-m}{\tau_2}\right)\left(\frac{m}{\tau_2}+1\right)\right]. \quad (8)$$

Also, by the same estimation procedure, we can estimate the above five parameters, $\varphi \equiv \{\beta_0, \beta_1, \beta_2, \tau_1, \tau_2\}$, embedded in Extended Nelson-Siegel model using the following nonlinear, constrained optimization estimation procedure based on the modified Gauss-Newton numerical method.

[1] The pricing error may be caused by transaction costs, coupon effects, market imperfection, and so on.

$$P_i = \sum_{m=1}^{T} \frac{CF_{im}}{\left\{1+\beta_0+\beta_1\left(\frac{\tau_1}{m}\right)\left[1-exp\left(\frac{-m}{\tau_1}\right)\right]+\beta_2\left(\frac{\tau_2}{m}\right)\left[1-exp\left(\frac{-m}{\tau_2}\right)\left(\frac{m}{\tau_2}+1\right)\right]\right\}^m}+\varepsilon_i \cdot \quad (9)$$

1.3. Nelson-Siegel-Svensson Model. To increase the flexibility and improve the fitting performance, Svensson (1994) extends Nelson and Siegel's instantaneous forward rate function by adding a fourth term, a second hump-shape (or U-shape), $\beta_3 \frac{m}{\tau_2} exp\left(-\frac{m}{\tau_2}\right)$, with two additional parameters, β_3 and τ_2. The function is then set as

$$f(m) = \beta_0 + \beta_1\, exp\left(\frac{-m}{\tau_1}\right) + \beta_2\left(\frac{m}{\tau_1}\right) exp\left(\frac{-m}{\tau_1}\right) + \beta_3\left(\frac{m}{\tau_2}\right) exp\left(\frac{-m}{\tau_2}\right), \quad (10)$$

where the unknown parameters β_0, β_1, β_2 and τ_1 have the same economic interpretation as the Nelson-Siegel model, and the two additional parameters, β_3 and τ_2 denote the same meaning as β_2 and τ_1.

The spot rate can be derived by integrating the forward rate and is given by

$$R(m) = \beta_0 + \beta_1\left(\frac{\tau_1}{m}\right)\left[1-exp\left(\frac{-m}{\tau_1}\right)\right] + \beta_2\left(\frac{\tau_1}{m}\right)\left[1-exp\left(\frac{-m}{\tau_1}\right)\left(\frac{m}{\tau_1}+1\right)\right] + \beta_3\left(\frac{\tau_2}{m}\right)\left[1-exp\left(\frac{-m}{\tau_2}\right)\left(\frac{m}{\tau_2}+1\right)\right]. \quad (11)$$

Similarly, in Eq. (12), the six parameters ($\varphi \equiv \{\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2\}$) embedded in Nelson-Siegel-Svensson Model could be obtained.

$$P_i = \sum_{m=1}^{T} \frac{CF_{im}}{\left\{1+\beta_0+\beta_1\left(\frac{\tau_1}{m}\right)\left[1-exp\left(\frac{-m}{\tau_1}\right)\right]+\beta_2\left(\frac{\tau_1}{m}\right)\left[1-exp\left(\frac{-m}{\tau_1}\right)\left(\frac{m}{\tau_1}+1\right)\right]+\beta\left(\frac{\tau_2}{m}\right)\left[1-exp\left(\frac{-m}{\tau_2}\right)\left(\frac{m}{\tau_2}+1\right)\right]\right\}^m}+\varepsilon_i . \quad (12)$$

1.4. Liquidity-weighted objective functions. As mentioned earlier, the pricing errors should be minimal if the term structure is the only factor that determines the price of a bond. Since the reliability of the term structure estimation heavily depends on the precision of market prices, as noted by Subramanian (2001), liquidity and illiquid securities are a heterogeneous class and including them both in the term structure estimation process poses problems. Illiquid bonds are traded at a premium to compensate for their undesirable attribute in terms of a lower price. Subramanian suggests a liquidity-weighted objective function, which hypothesizes that a weighted error function (with weights based on liquidity) would lead to better estimation than equal weights to the squared errors of all securities. More precisely, a hyperbolic tangent function (tanh) used to model the liquidity, which is represented by the volume traded in a security and the number of trades in that security, can be expressed as follows:

$$w_i = \left[tanh\left(-v_i / v_{max}\right)\right] + \left[tanh\left(-n_i / n_{max}\right)\right], \quad (13)$$

where v_i and n_i are the volume of trades and the number of trades in the *i* security, while $v_{\max}$ and $n_{\max}$ are the maximum volume of trades and the maximum number of trades for all the securities traded for the day. As given in Eq. (13), it ensures that the weights of the relatively liquid securities would not be significantly different from each other. For the illiquid securities, however, the weights would fall quickly as liquidity decreased.

In TGB market, the bonds are usually bought and sold in large lots, excluding transactions from small investors. Banks and other large financial institutions often purchase their positions in the primary market, and very rarely engage in the secondary market. Except for a few on-the-run issues, the transaction volumes in off-the-run issues are fairly small and illiquid. Thus, it is quite suitable to estimate the TGB term structure in a liquidity-weighted error function. In TGB market, the data set only contains the "the volume of trades" in a bond, however, "the number of trades" in each security is not available. Thus, with a simple modification, we can reset the liquidity-weighted error function as Eq. (14) owing to a positive relationship between "the volume of trades" and "the number of trades" in each security.

$$w_i = \left[tanh\left(-v_i / v_{max}\right)\right]. \quad (14)$$

The objective function is thus given by:

$$Min\sum_{i=1}^{n} w_i\left(P_i - B_i\right)^2 = Min\sum_{i=1}^{n} w_i\varepsilon_i^{\,2} . \qquad (15)$$

1.5. Test statistics. It is worth noting that, in the academic literature, there are two distinct approaches used to indicate the term structure fitting performance. One is the flexibility of the curve (accuracy), and the other focuses on the smoothness for the yield curve. Although there are numerical methods proposed to estimate the term structure, any method developed has to grapple with deciding the extent of the above trade-off. Hence, in the literature, it becomes a crucial issue to investigate how to reach a compromise between the flexibility and smoothness.

Three simple summary statistics which can be calculated for the flexibility of the estimated yield curve are the coefficient of determination, root mean squared percentage error, and root mean squared error. These are calculated as:

(1) The coefficient of determination (R^2)

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(P_i - \hat{B}_i)^2 /(n-k)}{\sum_{i=1}^{n}(P_i - \overline{P})^2 /(n-1)},$$

where $\overline{P}$ is the mean average price of all observed bonds, $\hat{B}_i$ is the model price of bond *i*, *k* is the number of parameters needed to estimate. Roughly speaking, with the same analysis in regression, we associate a high value of R^2 with a good fit of the term structure and associate a low value R^2 with a poor fit.

(2) Root mean squared percentage error (RMSPE)

$$RMSPE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{P_i - \hat{B}_i}{P_i}\right)^2} * 100\% .$$

Denoted as the RMSPE, a low value for this measure is assumed to indicate that the model is flexible, on average, and is able to fit the yield curve.

(3) Root mean squared error (RMSE)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(P_i - \hat{B}_i\right)^2} .$$

Denoted as the RMSE, a low value for this measure is also assumed to indicate that the model is flexible, on average, and is able to fit the yield curve.

Meanwhile, we use the following index, a modified statistic suggested by Adams and Deventer (1994) to reach the maximum smoothness for forward rate curve, and denote the smoothness (Z) for the estimated yield curve as:

$$Z = \sum_{t=1.5}^{15}\left(\left[f(t) - f(t-0.5)\right] - \left[f(t-0.5) - f(t-1)\right]\right)^2 \times 0.5 .$$

2. Empirical results

2.1. Data. In Taiwan, nearly all bond transactions take place on the OTC market. The Electronic Bond Trading System (EBTS) is now the main trading platform used by most securities firms and dealers for price negotiation (quoted in yields to maturity). The data used in this study are taken from the Taiwan Economic Journal Data Bank (TEJDB). The sample period contains 417 weekly data from January 2000 to December 2007. Weekly prices (every Friday) for 115 TGBs with original maturity dates ranged from 2 to 30 years are obtained. Although the TGBs are not actively traded, the actual sample sizes are more than 27 TGBs in each sample week.

2.2. Fitting performance without liquidity constraint. *2.2.1. Nelson-Siegel Model.* Table 1 lists the summary statistics of estimated parameters for the Nelson-Siegel Model. For the purposes of describing the movements of estimated yield curves more clearly, we divide our sample periods into seven sub-periods by years. It is obvious that, in most cases, we have a negative mean value ($\hat{\beta}_1$) and positive mean value ($\hat{\beta}_2$), which indicates that the yield curves in years 2000, 2002, 2003, 2004, and 2007 have a positively upward sloping and a slightly humped shape. In addition, in the years of 2001, 2005 and 2006, we have a negative mean value ($\hat{\beta}_1$ and $\hat{\beta}_2$), which indicates the yield curves have a positively upward sloping.

Figure 2 shows the movements of the estimated yield curves for the Nelson-Siegel Model. We estimate a discrete yield curve which is composed of 30 different maturities from 1 to 30 years. Generally speaking, the estimated yield curves for the three fitting models move from the same downward trend, especially in the long-term end. This pattern is also consistent with the tendency that the long-term interest rates in global economy have been especially prone to fall for the last two decades. However, in the short-term end, the phenomenon that estimation on short-term spot rates seems fluctuating could be on account of the trading observations for short-term TGBs being insufficient.

Table 1. Results of estimated parameters for Nelson-Siegel Model (without liquidity constraint)

Year	Parameters			
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}$
2000	0.0565	-0.0144	0.0060	3.1712
2001	0.0471	-0.0118	-0.0124	4.2369
2002	0.0385	-0.0155	0.0123	4.7833
2003	0.0346	-0.0205	0.0099	9.0127
2004	0.0339	-0.0187	0.0199	9.9274
2005	0.0263	-0.0132	-0.0095	2.5219
2006	0.0214	-0.0072	-0.0246	1.4887
2007	0.0232	-0.0081	0.0070	2.5831

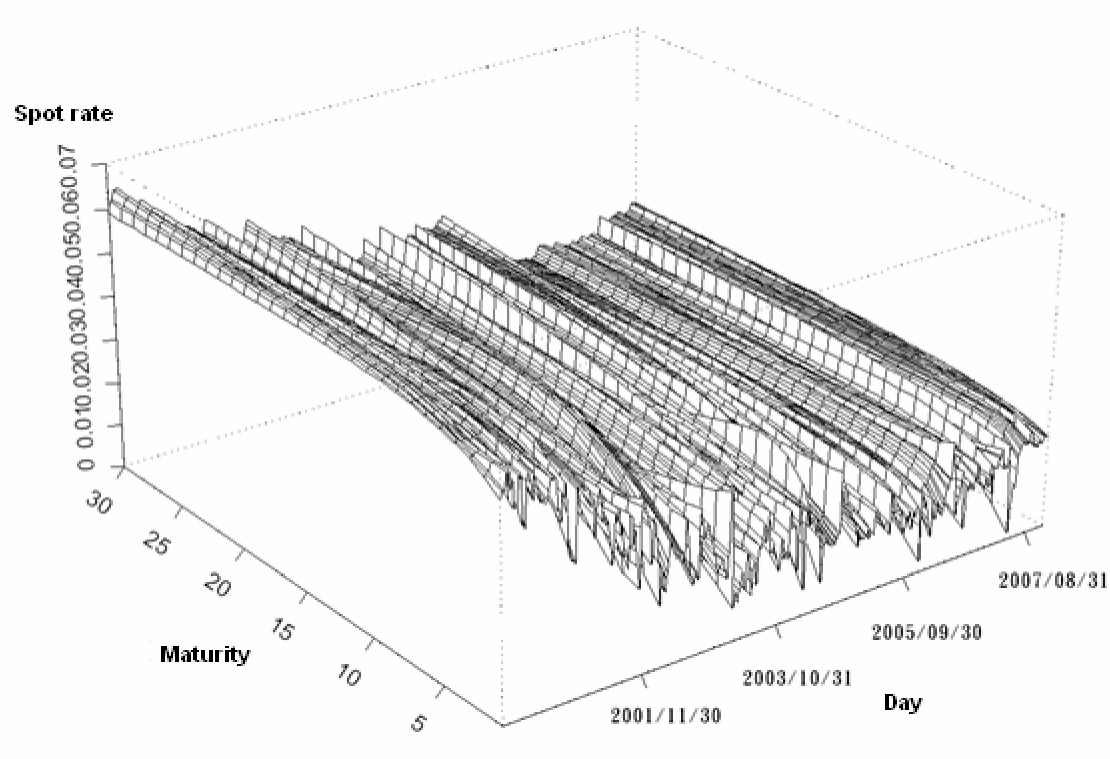

Fig. 2. The time paths of estimated yield curves for Nelson-Siegel Model (without liquidity constraint)

2.2.2. Extended Nelson-Siegel Model. Table 2 lists the summary statistics of estimated parameters for the Extended Nelson-Siegel Model. Figure 3 shows the movements of the estimated yield curves for the Extended Nelson-Siegel Model. Both the estimated parameters, $\hat{\beta}_1$ and $\hat{\beta}_2$, are negative in the years 2000, 2003 and 2005. It indicates that the yield curves are positively upward sloping. And in the years 2001, 2002, 2004, 2006 and 2007, the estimated $\hat{\beta}_1$ is negative and the estimated $\hat{\beta}_2$ is positive, showing the yield curves have a positively upward sloping and a slightly humped shape.

Table 2. Results of estimated parameters for Extended Nelson-Siegel Model (without liquidity constraint)

Year	Parameters				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}_1$	$\hat{\tau}_2$
2000	0.0598	-0.0179	-0.0010	1.7217	3.4570
2001	0.0462	-0.0162	0.0087	3.7184	4.3798
2002	0.0422	-0.0164	0.0011	4.4535	4.4226

Table 2 (cont.). Results of estimated parameters for Extended Nelson-Siegel Model (without liquidity constraint)

Year	Parameters				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}_1$	$\hat{\tau}_2$
2003	0.0366	-0.0195	-0.0197	3.8404	2.9491
2004	0.0317	-0.0179	0.0049	0.0273	11.9753
2005	0.0297	-0.0147	-0.0032	7.5070	7.2075
2006	0.0245	-0.0118	0.0035	9.3278	2.6375
2007	0.0242	-0.0174	0.0182	3.6225	1.5803

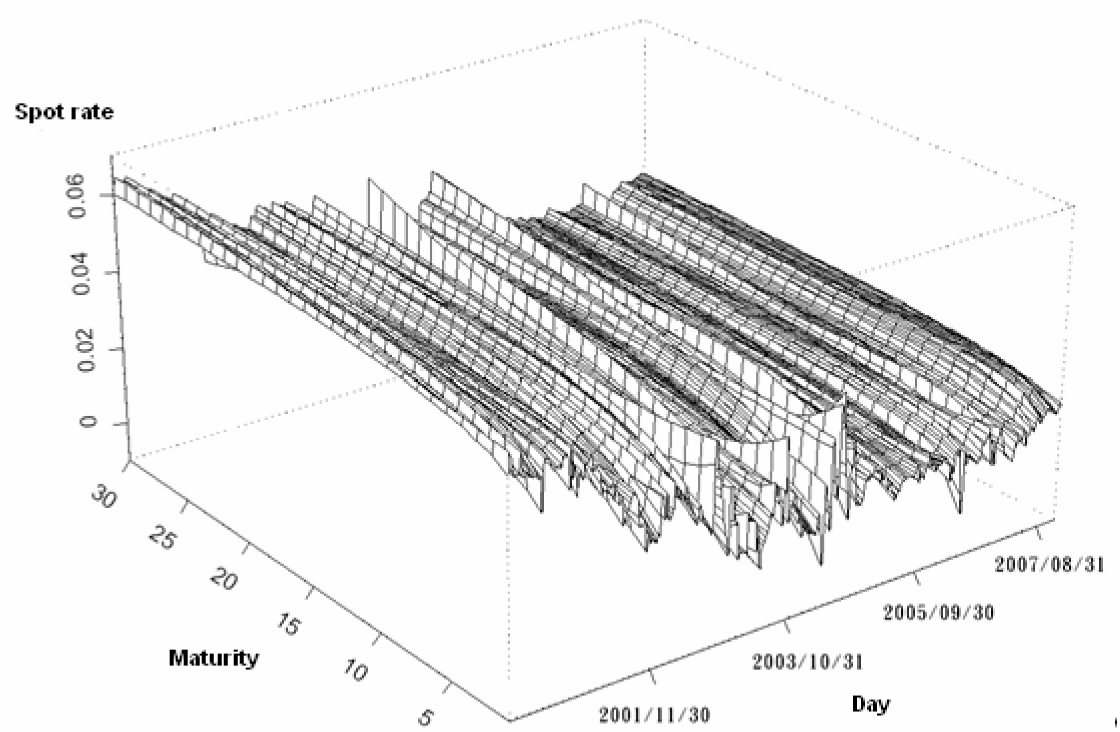

Fig. 3. The time paths of estimated yield curves for Extended Nelson-Siegel Model (without liquidity constraint)

2.2.3. Nelson-Siegel-Svensson Model. Table 3 lists the summary statistics of estimated parameters for the Nelson-Siegel-Svensson Model. Figure 4 shows the movements of the estimated yield curves for the Nelson-Siegel-Svensson Model. Both the estimated parameters, $\hat{\beta}_1$ and $\hat{\beta}_2$, are negative from the year of 2001 to 2006. It indicates that the yield curves are positively upward sloping. And in 2000, the estimated $\hat{\beta}_1$ is negative and the estimated $\hat{\beta}_2$ is positive, showing the yield curves have a positively upward sloping and a slightly humped shape. On the contrary, in the year 2007, the average estimated $\hat{\beta}_1$ value is positive and the average estimated $\hat{\beta}_2$ value is negative, which indicates the yield curves have a negatively downward sloping shape.

Thus, from the empirical results, we conclude that a different shape for estimated yield curve will be obtained if we adopt different fitting models to estimate the term structure of interest rates.

Table 3. Results of estimated parameters for Nelson-Siegel-Svensson Model (without liquidity constraint)

Year	Parameters					
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\tau}_1$	$\hat{\tau}_2$
2000	0.0595	-0.0103	0.0163	-0.0195	3.9425	3.8807
2001	0.0076	-0.0027	-0.0540	0.0080	13.0955	13.0886

Table 3 (cont.). Results of estimated parameters for Nelson-Siegel-Svensson Model (without liquidity constraint)

Year	Parameters					
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\tau}_1$	$\hat{\tau}_2$
2002	0.0441	-0.0253	-0.0265	-0.0013	1.1500	1.1258
2003	0.0356	-0.0270	-0.0077	-0.0111	2.1545	1.8582
2004	0.0351	-0.0199	-0.0134	0.0060	3.8562	2.8016
2005	0.0274	-0.0084	-0.0135	-0.0072	2.1134	2.0475
2006	0.0254	-0.0060	-0.0009	-0.0146	2.4073	2.4580
2007	0.0251	0.0084	-0.0170	-0.0138	1.5267	1.4968

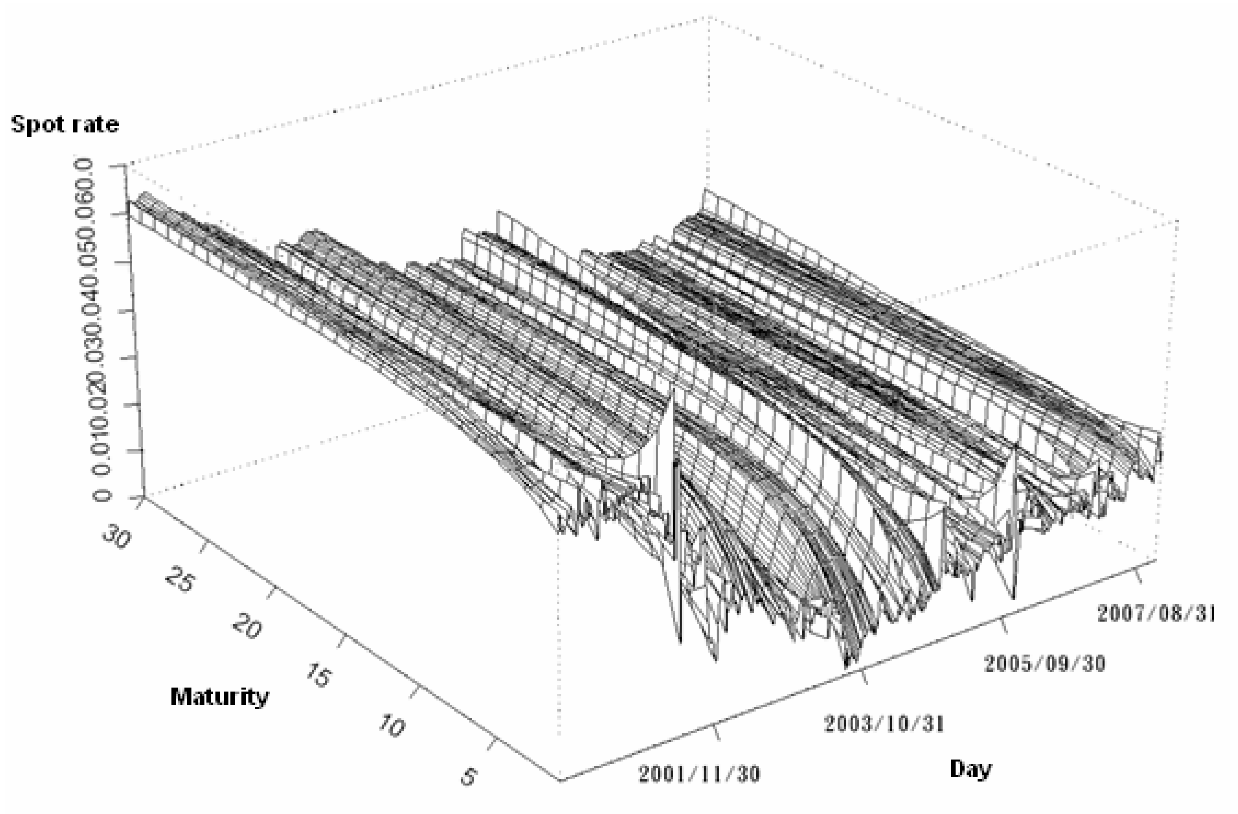

Fig. 4. The time paths of estimated yield curves for Nelson-Siegel-Svensson Model (without liquidity constraint)

2.2.4. Comparison for fitting performance in accuracy. As mentioned above, the shape of the yield curve is quite different using a family of Nelson-Siegel yield curve models. To move further ahead, we list the summary statistics in Table 4, the coefficient of determination, root mean squared percentage error, and root mean squared error, and compare the fitting performance for these three models. From Table 4, we find the R-square value of these three models is higher than 0.96. This shows the superiority of all these three models. The fitting performance of Nelson-Siegel-Svensson Model (0.9706) is better than that of Extended Nelson-Siegel Model (0.9646), and the Extended Nelson-Siegel Model is better than that of Nelson-Siegel Model (0.9638). Similarly, we can also reach the same conclusion if we adopt another two indicators, the RMSPE and RMSE values as judgment criteria. This means that adding a parameter can better capture the shape of the term structure in reality.

We conclude that, in Table 4, the Nelson-Siegel-Svensson Model has best fitting performance in accuracy from the mean value for three models and the differences in absolute value between these models seem significant. Here the statistics test for fitting performance in accuracy is displayed in Table 5, which indicates that at the 5% and 10% levels, the R^2 of Nelson-Siegel-Svensson model is higher than that of Nelson-Siegel model and Extended Nelson-Siegel model in terms of paired tests.

Table 4. Summary statistics for fitting performance in accuracy (without liquidity constraint)

	RMSPE			RMSE			R^2		
	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson
Mean	0.0145	0.0145	0.0129	1.6439	1.6420	1.4746	0.9638	0.9646	0.9706
Std. dev	0.0067	0.0072	0.0062	0.8085	0.8551	0.7410	0.0510	0.0498	0.0356
Max	0.0495	0.0494	0.0395	5.8012	5.8096	4.9366	0.9977	0.9982	0.9976
Min	0.0046	0.0049	0.0040	0.4945	0.5252	0.4275	0.8068	0.7982	0.7939

Table 5. Statistics test for fitting performance in accuracy (without liquidity constraint)

	R^2			RMSPE			RMSE		
	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel
Nelson-Siegel-Svensson	-	1.9295*	2.1925**	-	-3.2895***	-3.3734***	-	-3.1334***	-3.1042***
Extended Nelson-Siegel		-	0.2713		-	0.0375		-	0.1255
Nelson-Siegel			-			-			-

Notes: * denotes significance at 10% level, ** denotes significance at 5% level, *** denotes significance at 1% level.

2.3. Fitting performance with liquidity constraint. *2.3.1. Nelson-Siegel Model.* Table 6 lists the summary statistics of estimated parameters for the Nelson-Siegel Model with liquidity constraint. It is seen that all the estimated values for $\hat{\beta}_1$ and $\hat{\beta}_2$ are negative in seven sub-sample periods. It indicates that the yield curves are positively upward sloping. The movements of the estimated yield curves for the Nelson-Siegel Model with liquidity constraint are displayed in Figure 5. Compared with Figure 1, the estimated yield curves are much smoother. Thus, we conclude that it will provide diverse shapes in estimated yield curves for the same Nelson-Siegel Model while incorporating the liquidity constraint.

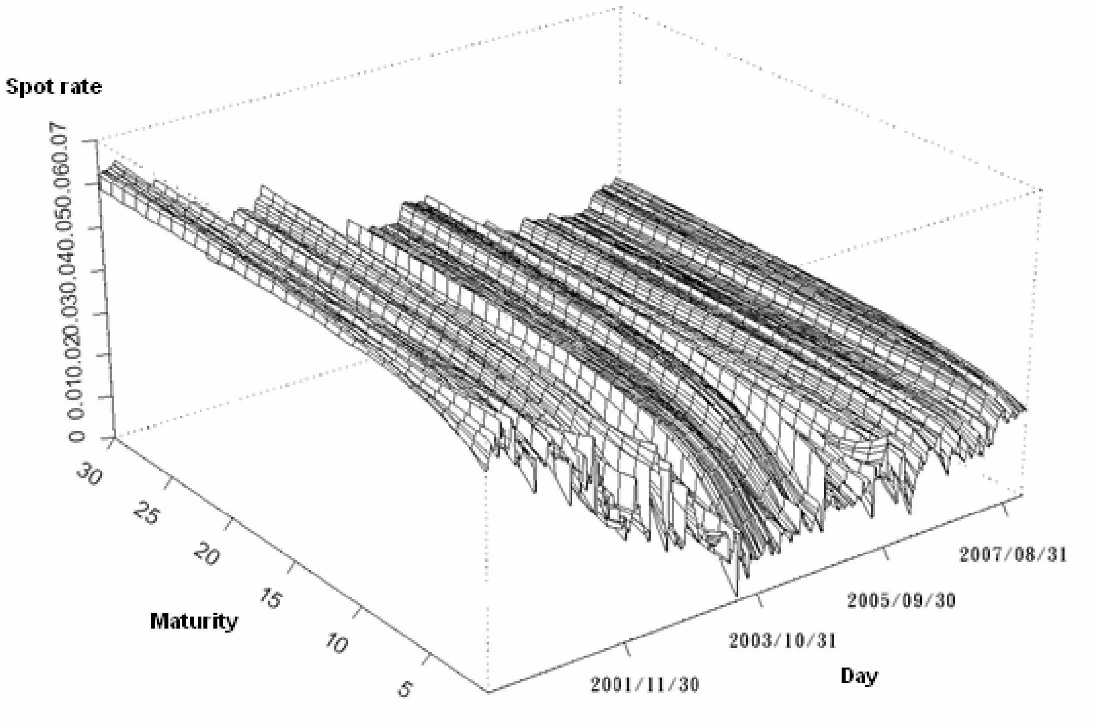

Fig. 5. The time paths of estimated yield curves for Nelson-Siegel Model (with liquidity constraint)

Table 6. Results of estimated parameters for Nelson-Siegel Model (with liquidity constraint)

Year	Parameters			
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}$
2000	0.0587	-0.0115	-0.0040	4.6089
2001	0.0463	-0.0090	-0.0127	3.2148
2002	0.0454	-0.0133	-0.0388	1.8327
2003	0.0356	-0.0183	-0.0425	2.1674
2004	0.0358	-0.0164	-0.0493	1.0232
2005	0.0225	-0.0085	-0.0819	0.6237
2006	0.0225	-0.0085	-0.0819	0.6237
2007	0.0241	-0.0091	-0.0213	1.0595

2.3.2. Extended Nelson-Siegel Model. Table 7 reports the summary statistics of estimated parameters for the Extended Nelson-Siegel Model. Figure 6 shows the movements of the estimated yield curves for the Extended Nelson-Siegel Model. We can find that, in the years 2002, 2005, 2006 and 2007, the estimated $\hat{\beta}_1$ is negative and the estimated $\hat{\beta}_2$ is positive, showing the yield curves have a positively upward sloping combined with a slightly humped shape; and the estimated yield curves for other sample periods show a positively upward sloping shape.

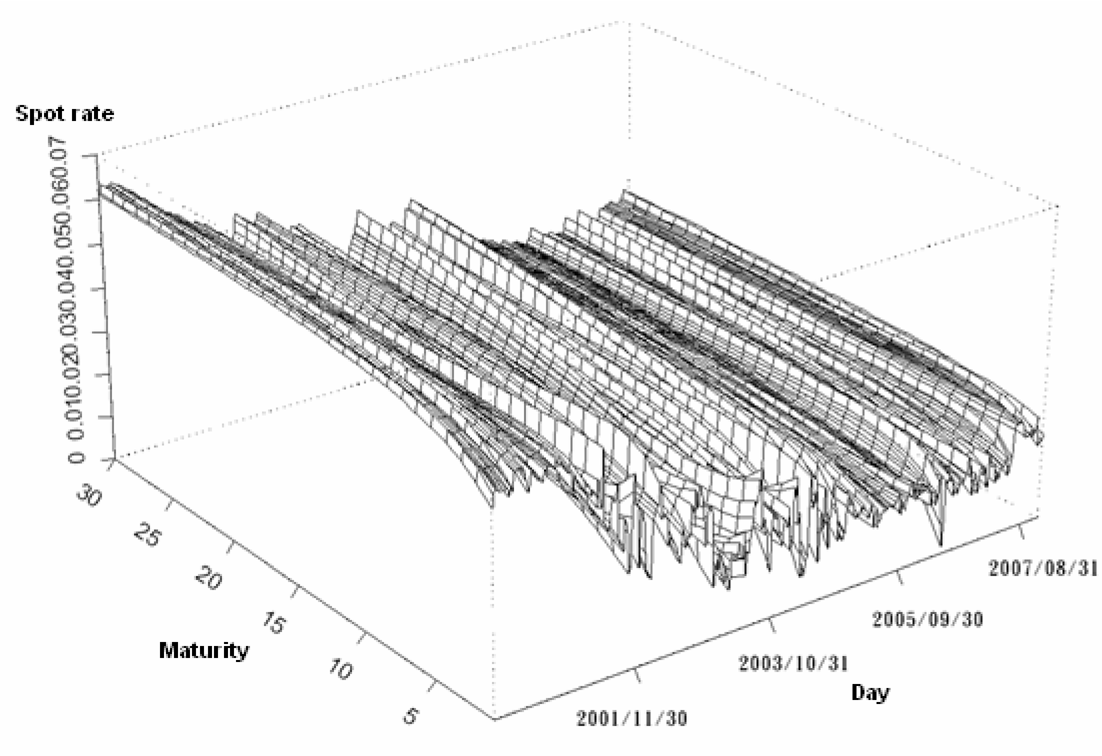

Fig. 6. The time paths of estimated yield curves for Extended-Nelson-Siegel Model (with liquidity constraint)

Table 7. Results of estimated parameters for Extended Nelson-Siegel Model (with liquidity constraint)

Year	Parameters				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}_1$	$\hat{\tau}_2$
2000	0.0590	-0.0138	-0.0010	2.1459	3.3983
2001	0.0473	-0.0121	-0.0068	2.5956	3.8261

Table 7 (cont.). Results of estimated parameters for Extended Nelson-Siegel Model (with liquidity constraint)

Year	Parameters				
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\tau}_1$	$\hat{\tau}_2$
2002	0.0425	-0.0182	0.0020	3.3964	5.4206
2003	0.0370	-0.0189	-0.0333	2.5649	8.1488
2004	0.0354	-0.0181	-0.0309	2.8238	7.6419
2005	0.0248	-0.0129	0.0025	8.0086	15.4666
2006	0.0248	-0.0129	0.0025	8.0086	15.4666
2007	0.0250	-0.0114	0.0072	3.3570	4.1933

2.3.3. Nelson-Siegel-Svensson Model. Table 8 lists the summary statistics of estimated parameters for the Nelson-Siegel-Svensson Model. Figure 7 shows the movements of the estimated yield curves for the Nelson-Siegel-Svensson Model. From the years 2000, 2002, and 2003 the estimated $\hat{\beta}_1$ is negative and the estimated $\hat{\beta}_2$ is positive, showing the yield curves have a positively upward sloping combined with a slightly humped shape. And both the estimated parameters, $\hat{\beta}_1$ and $\hat{\beta}_2$, are negative in the years 2001, 2004 to 2007, showing that the yield curves are positively upward sloping.

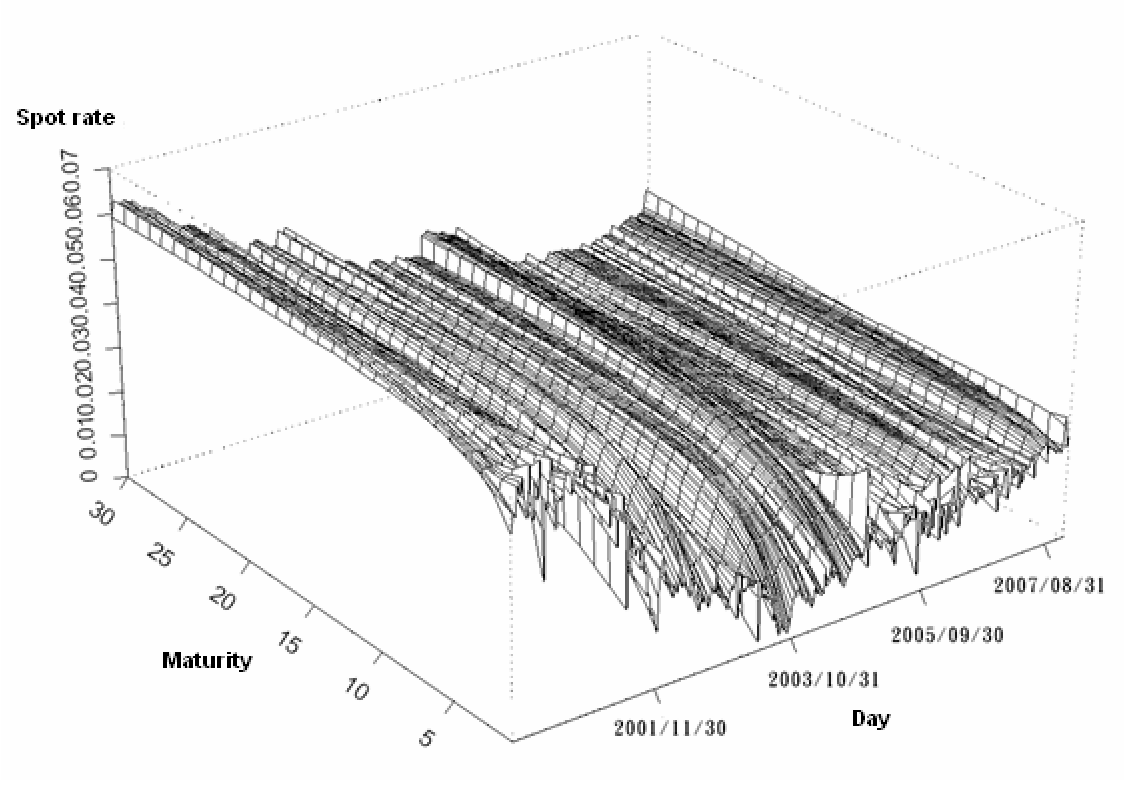

Fig. 7. The time paths of estimated yield curves for Nelson-Siegel-Svensson Model (with liquidity constraint)

Table 8. Results of estimated parameters for Nelson-Siegel-Svensson Model (with liquidity constraint)

Year	Parameters					
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\tau}_1$	$\hat{\tau}_2$
2000	0.0588	-0.0125	0.0316	-0.0365	2.4513	2.3036
2001	0.0477	-0.0152	-0.0034	0.0010	3.4306	3.4287
2002	0.0400	-0.0284	0.0701	-0.0410	3.8971	3.6562
2003	0.0355	-0.0293	0.0070	-0.0189	2.3245	2.1520

Table 8 (cont.). Results of estimated parameters for Nelson-Siegel-Svensson Model (with liquidity constraint)

Year	Parameters					
	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\tau}_1$	$\hat{\tau}_2$
2004	0.0359	-0.0308	-0.0068	0.0115	3.2297	2.9008
2005	0.0232	-0.0022	-0.0074	-0.0110	1.1180	1.1183
2006	0.0232	-0.0022	-0.0074	-0.0110	1.1180	1.1183
2007	0.0279	-0.0061	-0.0077	-0.0055	3.7633	3.7498

2.3.4. Comparison of fitting performance in accuracy. In contrast to the studies without considering the liquidity constraint, a direct comparison for the three models in Table 9 appears to favor the Nelson-Siegel-Svensson yield curve. It is further interesting to note that, from these three comparison indicators, we still reach the same conclusion for the ranking of their fitting performance. Looking at the statistics test in Table 10, formal statistics tests for three models, the Nelson-Siegel-Svensson Model does appear significantly better than the rest. The Nelson-Siegel Model, however, shows the worst fitting performance among the three models. Hence, we conclude that in the illiquid bond market, based on a family of Nelson-Siegel yield curve models, it does help to improve the flexibility of the yield curve if we add extra parameters in the parsimonious yield curve model.

Table 9. Summary statistics for fitting performance in accuracy (with liquidity constraint)

	RMSPE			RMSE			R^2		
	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson	Nelson-Siegel	Extended Nelson-Siegel	Nelson-Siegel-Svensson
Mean	0.0144	0.0131	0.0122	1.6318	1.4914	1.4043	0.9654	0.9693	0.9738
Std. Dev.	0.0066	0.0061	0.0051	0.7752	0.7299	0.6033	0.0357	0.0368	0.0299
Max	0.0413	0.0385	0.0281	4.6311	4.5345	3.3047	0.9973	0.9976	0.9979
Min	0.0050	0.0048	0.0041	0.5311	0.5109	0.4363	0.8015	0.7671	0.8219

Table 10. Statistics test for fitting performance in accuracy (with liquidity constraint)

	R^2			RMSPE			RMSE		
	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel	Nelson-Siegel-Svensson	Extended Nelson-Siegel	Nelson-Siegel
Nelson-Siegel-Svensson	-	1.9230*	3.6772***	-	-1.8756*	-4.7236***	-	-2.2599**	-5.2520***
Extended Nelson-Siegel		-	1.5555*		-	-2.6898***		-	-2.8491***
Nelson-Siegel			-			-			-

Notes: *** denotes significance at 1% level,** denotes significance at 5% level, * denotes significance at 10% level.

As mentioned earlier in the previous section, the empirical results report that the Nelson-Siegel-Svensson yield curve performs the best fitting performance, with and without the liquidity constraint, in terms of curve flexibility across the sample period. Thus, we only test the Nelson-Siegel-Svensson Model to compare the fitting performance on condition of whether the liquidity factor is considered or not. Table 11 shows the results of pair-wise tests for fitting performance in accuracy (R^2). The results in Table 11 demonstrate that it can raise the term structure fitting performance if the liquidity factor is included. In this study, it is also demonstrated that the liquidity-weighted objective functions proposed by Subramanian (2001) is computationally efficient and appropriate to fit the yield curves in TGB market.

Table 11. Results for pair-wise tests of fitting performance in accuracy

	With liquidity constraint	Without liquidity constraint
With liquidity constraint	-	1.4126*
Without liquidity constraint	-1.4126*	-

Note: 1. * denotes significance at 10% level, the Nelson-Siegel-Svensson model with liquidity constraint is higher than that of Nelson-Siegel model without liquidity constraint; 2. R^2 is used to stand for the fitting performance in accuracy.

2.4. Comparison for fitting performance in accuracy and smoothness. When comparing alternative methods of term structure fitting models, there is a trade-off between flexibility and smoothness. In Table 12, the Nelson-Siegel-Svensson Model seems to have the best fit in flexibility for fitting the term structure of TGB market. However, the improvement for flexibility of yield curve must be compensated by the expense of a decrease in smoothness, thus to the contrary, the Nelson-Siegel Model behaves the relatively smooth yield curve. Moreover, comparing the smoothness (*Z*) of these three models, the Nelson-Siegel model is superior to its counterparts, the Extended Nelson-Siegel and the Nelson-Siegel-Svensson Model. The possible explanation is the over-parameters problem for the latter two models. So, as noted by Bliss (1996), "term structure estimation is an art, the trade-off of fit against parsimony and judgment of what differences are materials will always be subjective and depend on the problem at hand."

Table 12. Statistics tests for fitting performance in accuracy and smoothness

	Without liquidity constraint		With liquidity constraint	
	R^2	Smoothness (Z)(x 10^{-6})	R^2	Smoothness (Z)(x 10^{-6})
Nelson-Siegel	0.9638	4.9246	0.9654	4.9822
Extended Nelson-Siegel	0.9646	5.3660	0.9693	6.3840
Nelson-Siegel-Svensson	0.9706	6.7525	0.9738	10.7467

Conclusion

The term structure of interest rates is the most important concept in pricing all fixed income securities when observing the evolutions of the term structure of interest rates. The well estimated term structure can help in making investment decisions, forecasting future interest rates and managing interest rate risk. Under the rapid growth of the global bond market, the research of how to estimate a fitted term structure has become a very important issue and captures both the academic and practical interests. The purpose of this paper is to use a family of Nelson-Siegel yield curve models for estimating the term structure of interest rates in TGB market. The TGB market is smaller and illiquid compared to other bond markets of developing countries where only about a handful of liquid bonds get traded in a day. Illiquid bonds must also be included in the term structure estimation procedure. To overcome the illiquidity constraint in TGB market, we attempt to estimate the parameters by the weighted parameter optimization. As suggested by Subramanian (2001), the weights have been assigned based on the liquidity of individual securities and ensure that liquid bonds in the market are priced with smaller errors than the illiquid bonds.

The empirical results indicate that: (1) If we do not consider the liquidity constraint, the indicators for fitting performance in accuracy (R-square value) for three models are higher than 0.96. This shows the superiority for the family of Nelson-Siegel yield curve models. (2) In general, the fitting performance in accuracy for Nelson-Siegel-Svensson Model is better than that of Extended Nelson-Siegel Model, and the Extended Nelson-Siegel Model is better than that of Nelson-Siegel Model. It means that adding more parameters will have better capability in describing the shape of the term structure. (3) Compared with the case of which the liquidity constraint is not taken into consideration, these three models will have a better fitting performance if the liquidity constraint is considered.

The need of related research around the term structure is definitely necessary and imperative. Meanwhile, the Taiwan capital market has become one of the most important markets in the Asia-Pacific area. Our research results can be helpful for the government authority to draft its monetary policy and have important implications for the bond investors.

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