Отримані рівняння нестаціонарного температургоно поля у мембрані і корпусі тензорезистивного сенсора тиску при швидкозмінному термовпливі. На підставі цих рівнянь досліджується вплив термонапружень і термодеформацій у пружному елементі на статичну та динамічну характеристики сенсора. В дослідженнях показується взаємозв'язаність процесів, що впливають на температурну залежність сенсора. Запропоновані способи зменшення впливу цих процесів

Ключові слова: сенсор тиску, мембрана, термонапруження, швидкозмінна температура, метрологічні характеристики

Получены уравнения нестационарного температурного поля в мембране тензорезистивного сенсора давления при быстропеременном термовлиянии. На основании этих уравнений исследуется влияние термонапряжений и термодеформаций в упругом элементе на статическую и динамическую характеристики сенсора, а также рассматривается влияние термодеформации корпуса сенсора на мембрану. В исследованиях показывается взаимосвязь процессов, которые влияют на температурную зависимость сенсора. Предложены способы уменьшения влияния этих процессов

Ключевые слова: сенсор давления, мембрана, термонапряжения, быстропеременная температура, метрологические характеристики

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STUDY OF THE INFLUENCE OF A FAST-CHANGING TEMPERATURE ON METROLOGICAL CHARACTERISTICS OF THE TENSORESISTIVE PRESSURE SENSOR

M. Tykhan
Doctor of Technical Sciences, Associate Professor*
E-mail: tykhanm@ukr.net
T. Repetylo
PhD, Assistant*
E-mail: tarasrepetylo@gmail.com
I. Dilay
Doctor of Technical Sciences, Associate Professor**
E-mail: divlv@ukr.net
V. Markovych*
E-mail: viktormarkovuch@gmail.com
*Department of precision mechanics devices***
Department of Automation and Computer Integrated Technologies*
***Lviv Polytechnic National University
S. Bandery str.,12, Lviv, Ukraine, 79013

1. Introduction

At present, a whole series of technological processes require pressure sensors that can work under conditions of fast-changing thermal influences with significant amplitudes [1–4]. Thus, in spacecraft and launch vehicles pressure measurement is accompanied by temperature influence on sensors in the range from –196 °C to +700 °C at the rate of temperature change on the order of 4,000–5,000 °C/s). In aviation engineering, temperature influences range from –60°C to +350 °C, and in maritime technology – from –50 °C to +120 °C [4].

Under such conditions, a temperature error in sensors can amount to tens of percent. However, technical systems in strategic industries require that temperature measurement error should remain within the accuracy class.

Therefore, it is an important task to study the influence of fast-changing temperature on metrological characteristics of the sensor, specifically, on the static characteristic, and one of dynamic characteristics –the transition characteristic.

2. Literature review and problem statement

Among the many types of pressure sensors, which are used for measurements under conditions of non-stationary thermal influences, a leading place is taken by tensoresistive sensors [2–4]. Today there is a great variety of devices for measuring pressure, however, the most effective under conditions of harsh temperatures are the metal thin-film tensoresistive pressure sensors.

In order to clarify the possibilities of application of such sensors under conditions of temperature influences, a number of studies were conducted. In general, these studies, based on the subject area, can be divided into two groups: the research into temperature influence on tensoresistors, and the studies that address the effect of temperature on the primary transducer–membrane. Results of the first group of research made it possible to devise methods of thermo-compensation, which significantly extended the capabilities of sensors. The following papers are the most important in tackling a problem of the influence of temperature on the "mechanical" part of the sensor.

Paper [5] proposes a technique of thermoprotective films on the membrane of a sensor in order to reduce the rate of thermal influence. Such a technique produces a positive effect; however, it does not fundamentally eliminate thermal stress and thermal deformation of the sensor's membrane. That is why the issue of precision properties of the sensor under conditions of thermal influence in general remains to be defined.

In order to use at instantaneous influence of temperature, the sensor reported in [6] was developed. This is achieved through the use of a specialized technology for the formation of a tensoresistive structure and specially designed mechanical structure of the sensor. It is unclear, however, whether a given solution would be effective for a lasting and non-stationary temperature; also remains unexplained is the issue on dependence of metrological characteristics of the sensor on thermal influence. First of all, it concerns the static and dynamic characteristics.

Study [7] reported research into temperature compensation when measuring pressure. It was proposed to use two measurement circuits. One circuit is for measuring a deformation of the sensor's membrane due to the joint effect of temperature and pressure; another one only due to temperature. The difference in signals from these circuits enables the thermocompensation of the measuring signal. It is unclear, however, how tensoresistors would be able to distinguish between thermal deformation and joint deformation caused by pressure and temperature. Thus, the issue of influence of the temperature non-stationarity on metrological characteristics of the sensor remains open.

In paper [8], authors analyze thermal stresses in a pressure sensor. Specifically, they considered the problem of thermal stresses that occurred as a result of thermal mismatch between the sensing element and the structure where it is placed. As a result, there may emerge the thermal deflection of the sensor's membrane. However, remains unexplained is the issue of dependence of the static and dynamic characteristics on thermal influence, specifically non-stationary.

In addition, the considered problem of thermal deflection is not associated with the thermal deflection that occurs as a result of the direct action of nonstationary temperature on the element.

Paper [9] tackles the problem of influence of nonstationary temperature on the accuracy of tensoresistive sensors. However, the authors did not address the problem of the influence of temperature on dynamic characteristics, as well as the problem of thermal deflection, as a factor of influence on the additional temperature error of sensor measurement.

The system of temperature compensation for pressure sensors is presented in [10]. This system uses a high-precision temperature sensor and a digital circuit to provide temperature compensation. Such a method makes it possible to significantly influence the accuracy of pressure measurements under conditions of thermal influence, but it is completely ineffective when it is necessary to compensate for the implications of non-stationary thermal influence. In other words, the problem of the dependence of static and dynamic characteristics on the fast-changing temperature remains unexplained.

A thermocompensation method, presented in [11], employs new capabilities of semiconductor technology and implies using additional polysilicon resistors with a negative coefficient of resistance (TCR). However, the results of a given study cannot be applied to eliminate the effect of impact on the sensor caused by the nonstationary temperature. Thus, the problem of the dependence of static and dynamic characteristics on the fast-changing and nonstationary temperature influence remains to be solved in order to meet today's requirements.

It is obvious that existing studies leave unexplored certain thermal and thermomechanical processes in an elastic transducer and in a casing of the sensor under a fast-changing thermal influence. Thus, it is still not understood the way the mentioned processes that occur at a fast-changing temperature may affect the static and dynamic characteristics.

3. The aim and objectives of the study

The aim of present study is to examine the influence of fast-changing temperature, specifically, on the static and, one of the major dynamic, transition characteristics.

To accomplish the aim, the following tasks have been set:

– based on the dependences that describe thermal processes in the sensor's membrane and its casing, to obtain dependences, which would describe thermal stresses and thermal deformations in these elements;

– to estimate the impact of thermal stresses and thermal deformations in the sensor's membrane on its metrological characteristics, specifically on the static and dynamic characteristics;

– based on the research to be conducted, to work out recommendations on minimizing the impact of thermal stresses and thermal deformations in the sensor's membrane.

4. Thermal and thermomechanical processes in the membrane and casing of a pressure sensor under non-stationary thermal influence

4. 1. Thermal processes in the membrane and casing of a pressure sensor under non-stationary thermal influence

Standard design of the tensoresistive pressure sensor is shown in Fig. 1.

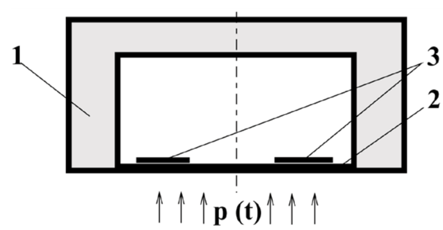

Fig. 1. Design of standard tensoresistive pressure sensor: 1 — casing, 2 — membrane, 3 — tensoresistors, $p(t)$ is the measured pressure

In this case, the membrane, which is a round plate with radius R and thickness h, is made of isotropic material, and has the following constants: coefficient of thermal conductivity R, specific heat capacity κ, coefficient of temperature conductivity

$$\chi = \frac{K}{\rho \cdot \kappa},$$

ρ is the density of membrane's material.

Under the action of the most fast-changing temperature, a thermal impact, the non-stationary temperature field in the membrane is described by equation [12]

$$T(r,z,t)=T_0+T_D(r,z,t)+T_y(r,z), \tag{1}$$

where

$$T_D(r,z,t)=-\frac{4\Delta T\cdot l_1\cdot l_3}{R}\times$$
$$\times\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Phi_2(z,\beta_m)\cdot e^{-\chi(a_n^2+\beta_m^2)t}\cdot\beta_m}{(a_n^2+\beta_m^2)\cdot\Phi_3(\beta_m)}\cdot\Phi_1(r,a_n) \tag{2}$$

– dynamic, or transitional, component,

$$T_y(r,z)=-\frac{2\Delta T\cdot l_1\cdot l_3}{R}\times$$
$$\times\sum_{n=1}^{\infty}\frac{a_n\operatorname{ch}(a_n(h-z))+l_2\operatorname{sh}(a_n(h-z))}{(l_1+l_2)a_n\operatorname{ch}(a_n h)+(l_1l_2+a_n^2)\operatorname{sh}(a_n h)}\Phi_1(r,a_n) \tag{3}$$

– constant component.

In these equations: ΔT is the amplitude of a thermal influence, hence

$$\Phi_1(a_n,r)=\frac{J_0(a_n r)}{(l_3^2+a_n^2)J_0(a_n R)}, \tag{4}$$

$$\Phi_2(\beta_m,z)=$$
$$=\beta_m\cos\beta_m(h-z)+l_2\sin\beta_m(h-z), \tag{5}$$

$$\Phi_3(a_n,r)=$$
$$=\beta_m\left[2+h(l_1+l_2)\right]\sin\beta_m h-$$
$$-\left[h\left(l_1\cdot l_2-\beta^2{}_m\right)+l_1+l_2\right]\cos\beta_m h, \tag{6}$$

where a_n are the roots of equation

$$a_n\cdot J_1(a_nR)-h_2\cdot J_0(a_nR)=0;$$

l_1, l_2, l_3 are the normalized heat exchange coefficients at the contact and inner surfaces and along the perimeter of a membrane; $J_0(a_nR)$ and $J_1(a_nR)$ are the Besel's functions; βm are the roots of equation

$$\operatorname{ctg}\beta_m h=\frac{1}{l_1+l_2}\left(\beta_m-\frac{l_1\cdot l_2}{\beta_m}\right);$$

T_1 is the temperature of the medium in a contact with the perimeter and inner surface; T_0 is the initial temperature of the membrane, which is the same all over its body (Fig. 2).

Casings of pressure sensors represent a massive (compared to a membrane) hollow cylinder (Fig. 1).

Under actual operation conditions, there occurs the heat exchange on all surfaces of the casing between media with variable temperatures (Fig. 3).

The non-stationary thermal field in this case will be described by equation

$$T(r,z,t)=\tilde{\Theta}+\sum_{i=1}^{\infty}A_i\cdot Z(\mu_i\cdot\zeta)\times$$
$$\times\left\{\begin{matrix}\hat{\Theta}+\exp(-p\mu_i{}^2\tau)\sum\limits_{n=1}^{\infty}B_n u_0(\beta_n\rho)\times\\ \times\left\{\Theta^*{}_0-\int\limits_0^{\tau}Q^*\cdot\exp\left[(\beta^2{}_n+p\mu_i{}^2)\tau'\right]d\tau'\right\}\exp-(\beta^2{}_n)\tau\end{matrix}\right\}. \tag{7}$$

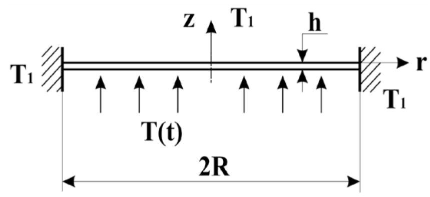

Fig. 2. Diagram of heat transfer at the surfaces of the membrane: *R* and *h* are the radius and thickness of the membrane; *T* is the temperature of the medium in a contact with the membrane's inner surface and internal perimeter; *T*(*t*) is the temperature of the measured environment; *r* and *z* are the coordinates along the radius and by the thickness of the membrane

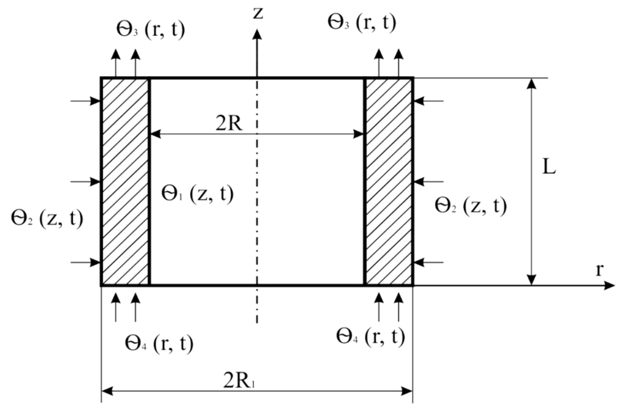

Fig. 3. Diagram of heat transfer on surfaces of the cylindrical casing of a pressure sensor

Under boundary and initial conditions

$$\left.\begin{aligned}&\frac{\partial T(r,z,t)}{\partial r}+l_1(T(r,z,t)-\Theta_1(z,t))=0\ \text{ at }\ r=R,\\&\frac{\partial T(r,z,t)}{\partial r}-l_2(T(r,z,t)-\Theta_2(z,t))=0\ \text{ at }\ r=R_1,\\&\frac{\partial T(r,z,t)}{\partial z}+l_3(T(r,z,t)-\Theta_3(r,t))=0\ \text{ at }\ z=L,\\&\frac{\partial T(r,z,t)}{\partial z}-l_4(T(r,z,t)-\Theta_4(r,t))=0\ \text{ at }\ z=0,\\&T(r,z,t)=\Theta_0=\text{const}\ \text{ at }\ t=0,\end{aligned}\right\} \tag{8}$$

where L is the height of the casing with inner radius R and outer radius R_1; l_1, l_2, l_3, l_4 are the normalized coefficients of heat exchange at the external and internal surfaces and at the ends of the cylinder; $\Theta_1(z,t),\Theta_2(z,t),\Theta_3(r,t),\Theta_4(r,t)$ are the temperatures of the media that are in contact with the outer and inner surfaces and the ends; Θ_0 is the initial temperature of the casing;

$$A_i=\frac{2\mu_i^2(\mu_i^2+\lambda_3^2)}{(\lambda_3+\lambda_4)(\mu_i^2+\lambda_3\lambda_4)+(\mu_i^2+\lambda_3^2)(\mu_i^2+\lambda_4^2)};\quad \kappa=\frac{r}{R_1};$$

$$Q^*=\int\limits_{\rho_1}^{1}\kappa\cdot Q^0u_0d\kappa;\quad Q^0=\bar{Q}-\left(\frac{d}{d\tau}+\varphi\cdot\mu_i{}^2\right)\hat{\Theta};$$

$$\bar{Q}=\int\limits_0^1 Q\cdot Z(\mu_i\cdot\zeta)d\zeta;\quad \Theta^*{}_0=\int\limits_{\rho_1}^{1}\kappa\cdot\Theta^0{}_0\cdot u_0d\kappa;$$

$$\Theta_0^0=\bar{\Theta}_0-\hat{\Theta}\Big|_{\tau=0};\quad \hat{\Theta}=\bar{\Theta}_1+\frac{\left(\bar{\Theta}_2-\bar{\Theta}_1\right)\cdot\lambda_2\cdot\kappa_1(1-\lambda_1\ln\kappa)}{\lambda_1+\lambda_2\cdot\kappa_1(1-\lambda_1\ln\kappa_1)};$$

$$\bar{\Theta}_k=\int_0^1 \Theta'_k \cdot Z(\mu_i \cdot \zeta)\mathrm{d}\zeta;\ \ (k=0,1,2).$$

$$\lambda_{3,4}=l_{3,4}\cdot L;\ \ \zeta=\frac{z}{L},\ \ \lambda_{1,2}=l_{1,2}\cdot R_1;\ \ \kappa_1=\frac{R}{R_1};\ \ Q=\Delta\tilde{\Theta};$$

$$\Delta=\frac{\partial^2}{\partial\kappa^2}+\frac{1}{\kappa}\frac{\partial}{\partial\kappa}-\frac{\partial}{\partial\tau};\ \ \tau=\frac{\chi\cdot t}{R_1^2};\ \ \varphi_1=\frac{R_1^2}{L^2};\ \ \Theta'_0=\Theta_0-\tilde{\Theta}\Big|_{\tau=0};$$

$$\Theta'_1=\Theta_1-\left(\frac{1}{\lambda_1}\frac{\partial}{\partial\kappa}+1\right)\cdot\tilde{\Theta}\Bigg|_{\kappa=1};\ \ \Theta'_2=\Theta_2+\left(\frac{1}{\lambda_2}\frac{\partial}{\partial\kappa}-1\right)\cdot\tilde{\Theta}\Bigg|_{\kappa=\kappa_1};$$

βn are the roots of equation

$$\frac{u'_0(\beta_n)}{u_0(\beta_n)}=\lambda_1;$$

μ_i are the roots of transcendental equation

$$tg\mu_i=\frac{\lambda_3+\lambda_4}{\mu_i^2-\lambda_3\lambda_4}\mu_i;$$

function $Z(\mu_i\cdot\zeta)$ is the solution to equation

$$\frac{d^2Z}{d\zeta^2}+\mu_i Z=0,$$

under conditions

$$\left.\begin{aligned}&\frac{dZ(\mu_i\cdot\zeta)}{d\zeta}+\lambda_3 Z(\mu_i\cdot\zeta)=0\ \text{ at }\ \zeta=1;\\&\frac{dZ(\mu_i\cdot\zeta)}{d\zeta}-\lambda_4 Z(\mu_i\cdot\zeta)=0\ \text{ at }\ \zeta=0;\end{aligned}\right\}$$

$$u_0(\beta_n\kappa)=\left[Y_1(\beta_n\kappa_1)+\frac{\lambda_2}{\beta_n}Y_0(\beta_n\kappa_1)\right]J_0(\beta_n\kappa)-$$
$$-\left[J_1(\beta_n\kappa_1)+\frac{\lambda_2}{\beta_n}J_0(\beta_n\kappa_1)\right]Y_0(\beta_n\kappa);$$

$$u'_0(\beta_n\kappa)=\left[Y_1(\beta_n\kappa_1)+\frac{\lambda_2}{\beta_n}Y_0(\beta_n\kappa_1)\right]J_1(\beta_n\kappa)-$$
$$-\left[J_1(\beta_n\kappa_1)+\frac{\lambda_2}{\beta_n}J_0(\beta_n\kappa_1)\right]Y_1(\beta_n\kappa).$$

The resulting equation (7) describes a non-stationary thermal field in the body of the sensor at the fastest change in a thermal influence, the thermal impact.

4. 2. Thermomechanical processes in the elements of the sensor, which are generated by non-stationary thermal fields

The non-stationary thermal field (1) will cause in the membrane the temperature stresses and thermal deformations that are described by dependences [13]

$$\sigma_r(r,z,t)=$$
$$=\frac{E}{1-\nu^2}\left[\varepsilon_r+\nu\varepsilon_\phi-(1+\nu)\lambda_m T(r,z,t)\right], \tag{9}$$

$$\sigma_\phi(r,z,t)=$$
$$=\frac{E}{1-\nu^2}\left[\varepsilon_\phi+\nu\varepsilon_r-(1+\nu)\lambda_m T(r,z,t)\right], \tag{10}$$

where λ_m is the coefficient of linear thermal expansion of membrane's material.

On the other hand, it is known [12] that the temperature difference for thickness, varying along the membrane's radius, will lead to a thermal deflection.

For the center of the membrane, such a thermal deflection is equal to

$$w_0(t,T^0)\big|_{r=0}=$$
$$=\sum_{n=0}^{\infty}\Phi_n(0)\cdot\frac{\zeta_n}{\eta_n}\int_0^t p_T(\tau)\cdot e^{-\beta(t-\tau)}\cdot\sin(\eta_n(t-\tau))\mathrm{d}\tau, \tag{11}$$

where $\Phi_n(0)=I_0(\mu_n)-J_0(\mu_n)$ are the eigenfunctions of the respective boundary value problem, $J_0(\mu_n)$, $I_0(\mu_n)$, $J_1(\mu_n)$, $I_1(\mu_n)$, $J_0(k_n r)$, $I_0(k_n r)$ are the Besel's functions,

$$\zeta_n=\frac{J_1(\mu_n)\cdot I_0(\mu_n)-J_0(\mu_n)\cdot I_1(\mu_n)}{\mu_n\cdot\gamma\cdot J_0^2(\mu_n)\cdot I_0^2(\mu_n)},$$

$\mu_n=k_n\cdot R$ are the eigenfunctions of the respective boundary value problem

$$\eta_n=\sqrt{\xi_n^2-\beta^2},\ \ \xi_n^2=c^4\cdot\frac{\mu_n^4}{R^4},\ \ c^4=\frac{D}{\gamma}=\frac{Eh^3}{12(1-\nu^2)\rho h},$$

$\gamma=\rho h$, E is the modulus of elasticity, ν is the Poisson coefficient, β is the damping coefficient of membrane oscillations,

$$p_T(t)=$$
$$=-\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{(1+\nu)D\lambda_m}{(1-\nu)h}\left[T(r,h,t)-T(r,0,t)\right]\right).$$

Expression (11) describes a thermal deflection of the membrane under condition of a sharp non-stationary thermal influence.

For example, at a thermal impact of amplitude $\Delta T=$ $=400$ °C, and with membrane parameters: $R=0.005$ m, $h=$ $=0.00025$ m, the thermal deflection of its center is 5.5 µm.

As it is known [12], at a given thermal deflection the values of relative deformations on the surface of the membrane can reach the level of working deformations of tensoresistors located on the membrane. This means that this very component of possible thermal deformation of the membrane will lead to a temperature error, which can approach a hundred percent.

Because the membrane is rigidly fixed in a massive casing, it cannot deform in its plane to a specific moment in time. Thus, in (9) and (10), ε_r, ε_ϕ are equal to zero, hence the thermal stresses will reach

$$\sigma_r(r,z,t)=\sigma_\phi(r,z,t)=$$
$$=\frac{E}{1-\nu}\left[-\lambda_m T(r,z,t)\right]. \tag{12}$$

Equivalent to thermal stresses (12), the radial efforts in the plane of the membrane will be equal to

$$N_r(t)=\frac{-E\lambda_m}{(1-\nu)}\frac{1}{R}\int_0^R\int_0^h T(r,z,t)\mathrm{d}z\mathrm{d}r. \tag{13}$$

If compressing (stretching) efforts $N(t)$ act in the plane of the rigidly fixed membrane, oscillations in the measurement of pressure $p(t)$ will be described by equation

$$c^4\Delta^2 w(r,t)\pm\frac{N(t)}{\gamma}\Delta w(r,t)+$$
$$+\frac{\partial^2 w(r,t)}{\partial t^2}+2\beta\frac{\partial w(r,t)}{\partial t}=\frac{p(t)}{\gamma}. \quad (14)$$

Solution (14) to the center of the membrane (r=0) will be recorded

$$w_0(t)=w(0,t)=$$
$$=\sum_{n=1}^{\infty}\Phi_n(0)\cdot\frac{\zeta_n}{\tilde{\eta}_n}\int_0^t p(\tau)\cdot e^{-\beta(t-\tau)}\cdot\sin(\eta_n(t-\tau))d\tau, \quad (15)$$

where

$$\tilde{\eta}_n=\sqrt{\tilde{\xi}_n^2-\beta^2};\quad \tilde{\xi}_n^2=c^4\cdot\frac{\mu_n^4}{R^4}\pm\frac{N(t)}{\gamma}\frac{\mu_n^2}{R^2}.$$

It is possible, for the casing of a sensor, using the resulting expression for a temperature field in the wall of the casing (7), to calculate the axial and radial thermal displacements of points on the inner wall of the casing (ΔR, ΔL, Fig. 4).

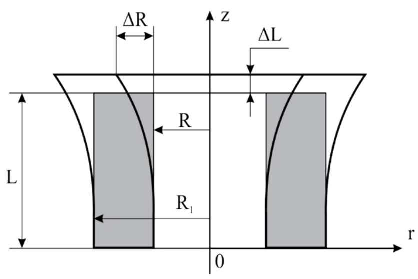

Fig. 4. Axial ΔL and radial ΔR thermal deformations of the sensor's casing: L, R, R_1 are the height, inner and outer radii of the casing

For example, at heat exchange, in the internal cavity of the casing filled with air medium of constant temperature, the absolute radial thermal displacements of points on the inner wall of the casing demonstrate dynamics shown in Fig. 5. We accepted the following parameters in the calculation: R=2 mm, R_1=2.5 mm, L=3 mm.

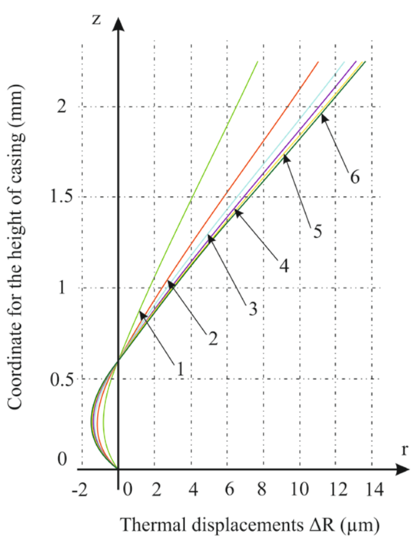

Fig. 5. Thermal displacements of the wall of the casing of a pressure sensor in the radial direction at discrete moments of time: 1 – at t=1 ms; 2 – at t=3 ms; 3 – at t=10 ms; 4 – at t=0.1 s; 5 – at t=1 s; 6 – at t=5 s

The modeling that we performed revealed that the relative thermal deformation of the casing in the region where the membrane is fixed was 0.7 %. It is this phenomenon that may exert a significant impact on the elastic element. Moreover, depending on the dynamics of thermal deformation of the casing, a membrane may at first compress, then stretch.

5. Analysis of the obtained results of studying the thermal and thermomechanical processes in the sensor's elements

If one assumes $p(\tau)=1$, in (15), we obtain a transitional characteristic of the sensor under the action of thermal stresses in the plane of membrane

$$\hbar_\sigma(t)=\sum_{n=0}^{\infty}\Phi_n(0)\cdot\frac{\zeta_n}{\tilde{\eta}_n}\times$$
$$\times\left[\frac{\tilde{\eta}_n-e^{-\beta t}(\tilde{\eta}_n\cdot\cos\tilde{\eta}_n t+\beta\sin\tilde{\eta}_n t)}{\tilde{\eta}_n{}^2+\beta^2}\right]. \quad (16)$$

Assuming $p(\tau)=p=\text{const}$ and $t=\infty$, in (15), we obtain a static characteristic under the action of thermal stresses in the plane of membrane

$$w(p)=\sum_{n=0}^{\infty}\Phi_n(0)\cdot\frac{\zeta_n}{\tilde{\xi}_n^2}\cdot p. \quad (17)$$

If one assumes $N(t)=0$, in (16), we obtain a transitional function under normal conditions ($\Delta T=0$)

$$\hbar(t)=\sum_{n=0}^{\infty}\Phi_n(0)\cdot\frac{\zeta_n}{\eta_n}\times$$
$$\times\left[\frac{\eta_n-e^{-\beta t}(\eta_n\cdot\cos\eta_n t+\beta\sin\eta_n t)}{\eta_n{}^2+\beta^2}\right]. \quad (18)$$

Fig. 6 shows: $\hbar(t)$ – transitional characteristic of the membrane under normal conditions, and $\hbar_\sigma(t)$ – under the action of thermal stresses, resulting from a thermal impact of amplitude ΔT=400 °C to the membrane with R=0.005 m and h=0.00025 m.

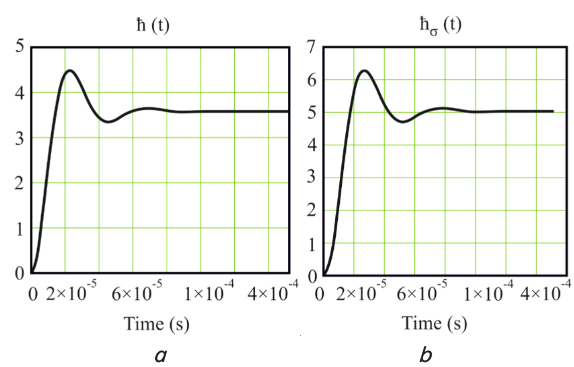

Fig. 6. Transitional characteristics of membrane: a – under normal conditions; b – under the action of thermal stresses

The difference between a transitional characteristic under the action of thermal stresses and under normal conditions is estimated by a relative error

$$\delta_\sigma(t)=\left(\frac{\hbar(t)-\hbar_\sigma(t)}{\hbar(t)}\right)100\,\%,$$

which is shown in Fig. 7.

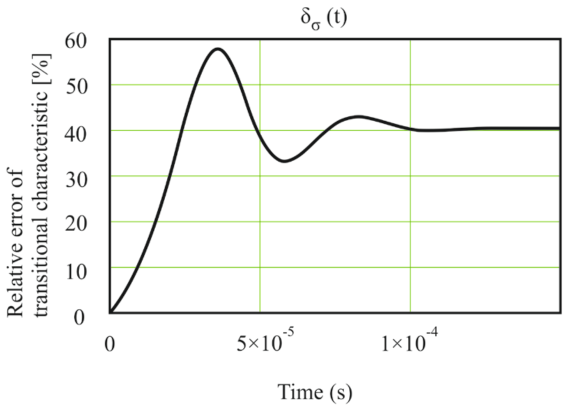

Fig. 7. Relative error of transitional characteristic caused by the action of thermal stresses

If the membrane could thermally deform in its plane (sliding pinching), then the relative value of such a deformation would equal to

$$\varepsilon_r(r,t) = (1+\nu)\lambda_m \times$$
$$\times\left[T(r,h/2,t) - \frac{1}{r^2}\int_0^r T(r,h/2,t)\mathrm{d}r - \frac{1}{R^2}\int_0^R rT(r,h/2,t)\mathrm{d}r\right]. \quad (19)$$

The dynamics of a given thermal deformation and values on the perimeter of the membrane with R=0.005 m and h=0.00025 m at a thermal impact of ΔT=400 °C are shown in Fig. 8.

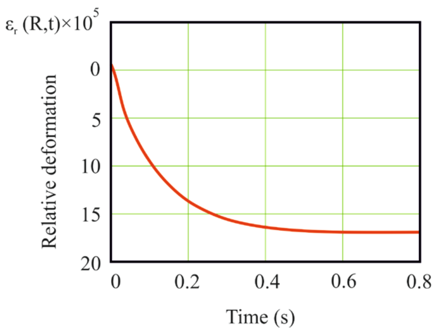

Fig. 8. Relative radial thermal deformation along a perimeter of the pinched membrane at thermal impact

The maximum value of relative radial thermal deformation of the membrane $\varepsilon_{r\max}$ reaches 1.7×10^{-4}, which is comparable to the working deformations of tensoresistors [14, 15]. Assuming $R = R_0(1+\varepsilon_{r\max})$, in (18), we obtain transitional characteristic $\hbar_\varepsilon(t)$ of the sensor for the membrane that was thermally deformed in its plane (Fig. 9, *a*). The deviation of a given characteristic from the normal characteristic of this will be also estimated by relative error

$$\delta_\varepsilon(t) = \left(\frac{\hbar(t) - \hbar_\varepsilon(t)}{\hbar(t)}\right)100\,\% \text{ (Fig. 9, } b\text{).}$$

With respect to (15), we obtain the formula for a static deflection of the membrane at the center under the action of pressure and compressing, or stretching, efforts $\pm N$ in its plane. Assuming $p(\tau)$=p_0 and t=∞ in (15), we obtain

$$w_0 = p_0 \cdot \frac{\left(J_1(\mu_0)\cdot I_0(\mu_0) - J_0(\mu_0)\cdot I_1(\mu_0)\right)\left(I_0(\mu_0) - J_0(\mu_0)\right)R^4}{J_0^{\,2}(\mu_0)\cdot I_0^{\,2}(\mu_0)\left(c^4\mu_0^5\gamma \pm N\mu_0^3R^2\right)}. \quad (20)$$

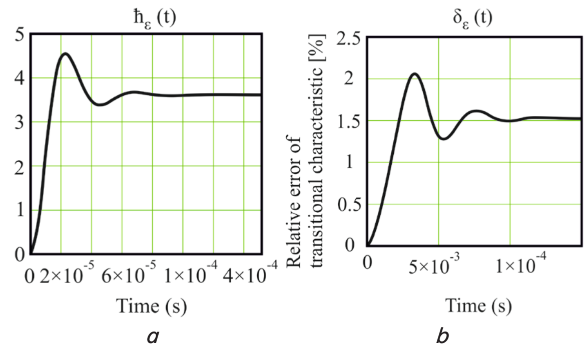

Fig. 9. Effect of thermal expansion on transitional characteristic of the membrane: *a* — transitional characteristic of the membrane that underwent thermal expansion; *b* — relative error of transitional characteristic

In other words, at a growth of compressing efforts N, a given deflection can constantly grow, and with the growth of stretching efforts, the deflection can accept a zero value. This means that the static characteristics of the membrane under conditions of a non-stationary thermal influence may remain undefined.

6. Discussion of results of studying the influence of fast-changing temperature on metrological characteristics of the tensoresistive pressure sensor

The results obtained show that the fast-change nature of temperature influence generates a spatial gradient of temperature field in the sensor's membrane. In turn, the gradient of temperature field generates a thermal deflection and compression or stretching of the membrane. In this case, each of these phenomena can cause a significant error (of the order of 60 %), in both static and transitional characteristics of the sensor. The numerical modelling that we performed shows that the relative deformations due to a thermal deflection can reach the level of working deformations of tensoresistors. In the absence of a possibility of deformation of the membrane in its plane, thermal stresses can distort the transitional characteristic of membrane up to 60 %. This is the reason why measurement errors of varying pressure under conditions of non-stationary temperatures are too large (of the order of 60 % and above).

In turn, under a "hot" thermal influence, a totality of the thermal deflection and compression stresses can cause a loss of stability of the membrane and its destruction. It is clear that under such circumstances a discussion of metrological characteristics makes no sense. On the other hand, at a "cold" thermal influence, the totality of the thermal deflection and stretching stresses may lead to the effect of apparent insensitivity of the membrane to the working pressure. In this case, however, both static and dynamic characteristics can have a significant error, and the destruction of the membrane may occur.

To minimize, and ideally eliminate, the thermal deflection, is possible by minimizing the gradient of temperature field along the radius of the membrane through thermal insulation of the perimeter, or via a special choice of parameters and materials for the sensor's casing and the membrane, which would possess appropriate thermal conductivities.

The analytical dependences that we obtained in the course of present research make it possible to estimate a possible thermal deflection of the pressure sensor's membrane based on actual operation conditions.

On the other hand, our findings show that in case we could manage to achieve a thermal "disclosure" of the sensor's casing in the radial direction, comparable and synchronous with thermal expansion of the membrane, it would contribute to a significant reduction of thermal stresses. It is obvious that such a sliding pinching of the membrane can be achieved through special design solutions of the sensor's body. Numeric modeling demonstrates that the thermal impact with a rather high amplitude (ΔT=400 °C), which caused relative thermal deformation to the membrane of the order of 10^{-4}, a distortion of the dynamic characteristic amounted to only 2 %. It is obvious that such a change is insignificant compared to a sixty-percent change in the transitional characteristic of a thermally stressed membrane. However, the thermal expansion (compression) of the order of 10^{-4} is essential for tensoresistors that are located on the membrane and, therefore, also causes a substantial temperature error.

The theoretical study conducted is of practical importance as it opens up ways for the non-stationary thermal influences, which would imply rapid adjustment of the temperature component of measurement results. These methods should include the calculation of the additional temperature error of the sensor caused by thermal-elastic phenomena in its membrane.

In general, the research performed shows that the maximum total error of pressure measurement under conditions of a rapidly-changing thermal influence can outreach 100 %, which agrees with the experience of measurements. Therefore, an analysis of the components of such an error could help in the development of technical solutions to reduce the error.

The results obtained will be particularly valuable in the design of pressure sensors for automatic control systems that require a quick adjustment of additional temperature errors of measurement.

It should be noted that when modeling thermal and thermomechanical processes, it is necessary to take into account a possible change in the properties of materials under the action of temperature, especially rapidly-changing. It is assumed that these properties would be constant. Such an assumption for the case of practical application of research results will require periodic validation of the sensors' metrological characteristics.

The reported results of research are theoretical in nature. Therefore, in the further research it is advisable to experimentally verify possible methods for eliminating the thermal deflection, as well as to develop design methods to minimize thermal stresses in the sensor's membrane.

7. Conclusions

1. We have obtained analytical dependences that describe a non-stationary temperature field in the membrane and cylindrical casing of the tensoresistive pressure sensor, which make it possible to analyze the character of the field and detect its patterns depending on specific parameters of the membrane and the casing.

2. It was established that the existence of a gradient along the radius of the membrane generates its thermal deflection, which significantly affects the accuracy of the sensor, since it may cause relative deformations on the surface of the membrane, which are commensurate to the working deformations (~10^{-4}) and thus may form a 100-percent additional temperature error.

3. It was found that the rapidly-changing thermal influence is the cause of a significant error in the static and transitional characteristics of the sensor (up to 60 %).

The most effective elimination of the negative influence of a fast-changing temperature on the precision characteristics of the sensor can be attained at the design level. That is, the very structure of elastic elements and the casing should have such a thermal-mechanical interaction that would minimize the negative effect of a rapidly-changing temperature.

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Запропоновано спосіб виготовлення у невагомості металевого сіткополотна за допомогою коливань ряду подвійних маятників. Коливання виникають завдяки впливу на вузли елементів маятника імпульсів двох реактивних двигунів, тим самим забезпечуючи його інерційне розкриття. Опис процесу інерційного розкриття маятника виконано за допомогою рівняння Лагранжа другого роду. Результати доцільно використати при проектуванні масштабних сіткополотен, наприклад активних поверхонь антен довгохвильового діапазону, та їх виготовлення в умовах невагомості

Ключові слова: геометричне моделювання, сіткополотно, подвійний маятник, розкриття антени, рівняння Лагранжа другого роду

Предложен способ изготовления в невесомости металлического сетеполотна при помощи колебаний ряда двухзвенных маятников. Колебания возникают благодаря влиянию на узлы элементов маятника импульсов двух реактивных двигателей, тем самым обеспечивая его инерционное раскрытие. Описание процесса инерционного раскрытия маятника выполнено с помощью уравнения Лагранжа второго рода. Результаты целесообразно использовать при проектировании масштабных сетеполотен, например активных поверхностей антенн длинноволнового диапазона, и их изготовления в условиях невесомости

Ключевые слова: геометрическое моделирование, сетеполотно, двойной маятник, раскрытие антенны, уравнение Лагранжа второго рода

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GEOMETRICAL MODELING OF THE PROCESS OF WEAVING A WIRE CLOTH IN WEIGHTLESSNESS USING THE INERTIAL UNFOLDING OF A DUAL PENDULUM

L. Kutsenko
Doctor of Technical Sciences, Professor
Department of Engineering and Rescue Technology*
E-mail: leokuts@i.ua
O. Semkiv
Doctor of Technical Sciences, Vice-Rector
Department of prevention activities and monitoring*
L. Zapolskiy
PhD, Senior Researcher
Scientific and organizational department
The State Emergency Service of Ukraine
Rybalska str., 18, Kyiv, Ukraine, 01011
O. Shoman
Doctor of Technical Sciences, Professor, Head of Department**
A. Kalynovskyi
PhD, Associate Professor
Department of Engineering and Rescue Technology*
M. Piksasov
PhD
Information Technology Center*
I. Adashevska
PhD, Associate Professor**
I. Shelihova
PhD, Associate Professor**
O. Sydorenko
PhD, Associate Professor**
*National University of Civil Protection of Ukraine
Chernyshevska str., 94, Kharkiv, Ukraine, 61023
**Department of Geometric Modeling and Computer Graphics
National Technical University "Kharkiv Polytechnic Institute"
Kyrpychova str., 2, Kharkiv, Ukraine, 61002

1. Introduction

Modern trends in the development of space systems for telecommunication necessitate creation of large and highly efficient antennas [1]. In most cases, reflector antennas are shaped in the form of a rigid frame that can be transformed and thus take on the calculated design form. The frame holds a flexible radio-reflective surface, which is made of metal