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FORMATION AND OPTIMIZATION OF VARIOUS PORTFOLIOS MODELS ON THE VaR INDICATOR BASIS

Abstract. This article describes a formation of various portfolios models based on H. Markovitz portfolio theory. The portfolios which can include instruments with fixed profitability and common stock are considered. As a risk measure VaR indicator is used. In the research historical data on the stock prices included in the Dow30 market index were used. The profitability of all shares was analyzed in the period from 01.09.2015 to 01.09.2016 on one day basis. Companies included in the surveyed portfolios were selected using factor analysis method. They are considered as a portfolio consisting of a uniform distribution of shares and optimization portfolios.

In these optimization models maximization of portfolio efficiency at the set risk level and risk minimization at the set level of efficiency can act as criterion functions. In some optimization tasks efficiencies of risky assets are calculated considering market changes. For portfolios of different profitability, the optimal curves "Profitability-Risk" are constructed. Comparison of the results is providing. Portfolios consisting of shares of various companies are explored. Depending on the investor's attitude to risk, appropriate portfolios can be chosen.

Numerical results of effective assets distribution within the portfolio for various optimization problems statements are shown.

Keywords: portfolio, risk, VaR, asset, optimization.

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ПОБУДОВА І ОПТИМІЗАЦІЯ МОДЕЛЕЙ РІЗНИХ ПОРТФЕЛІВ НА ОСНОВІ ПОКАЗНИКА VaR

Анотація. У даній роботі розглядається побудова різних моделей портфелів, грунтуючись на портфельної теорії Г. Марковіца. Розглядаються портфелі, складовими яких можуть бути акції та інструменти з фіксованою дохідністю. В якості міри ризику використовується показник VaR. Для аналізу використовувалися історичні дані цін акцій, що ввійшли до ринковий індексу Dow30. Проаналізовано прибутковості всіх акцій в період з 01.09.2015 по 01.09.2016 з інтервалом один день. Із застосуванням методу факторного аналізу, були відібрані компанії, які увійшли в досліджувані портфелі. Розглядаються як портфель, що складається з рівномірного розподілу акцій. так і оптимізаційні портфелі.

У побудованих оптимізаційних задачах цільовими функціями виступають: максимізація ефективності портфеля при заданому рівні ризику і мінімізації ризику при заданому рівні ефективності. В окремих оптимізаційних задачах знаходяться ефективності ризикових активів з урахуванням зміни ринку. Для портфелів різної прибутковості, побудовані оптимальні криві «Прибутковість-Ризик». Робиться порівняння отриманих результатів. Досліджуються портфелі, що складаються з акцій різних компаній. Залежно від ставлення інвестора до ризику можна вибрати відповідні портфелі.

Наводяться чисельні результати ефективного розподілу активів всередині портфеля для різних постановок оптимізаційних задач.

Ключові слова: портфель, ризик, прибутковість, VaR, актив, оптимізація

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ПОСТРОЕНИЕ И ОПТИМИЗАЦИЯ МОДЕЛЕЙ РАЗЛИЧНЫХ ПОРТФЕЛЕЙ НА ОСНОВЕ ПОКАЗАТЕЛЯ VaR

Аннотация. В данной работе рассматривается построение различных моделей портфелей, основываясь на портфельной теории Г. Марковица. Рассматриваются портфели, составляющими которых могут быть акции и инструменты с фиксированной доходностью. В качестве меры риска используется показатель *VaR*. Для анализа использовались исторические данные цен акций, включенных в рыночный индекс Dow30. Проанализированы доходности всех акций в период с 01.09.2015 по 01.09.2016 с интервалом один год. С применением метода факторного анализа, были отобраны компании, которые вошли в исследуемые портфели. Рассматриваются как портфель, состоящий из равномерного распределения акций, так и оптимизационные портфели.

В построенных оптимизационных задачах целевыми функциями выступают: максимизация эффективности портфеля при заданном уровне риска и минимизации риска при заданном уровне эффективности. В отдельных оптимизационных задачах находятся эффективности рисковых активов с учетом изменения рынка. Для портфелей различной доходности, построены оптимальные кривые «Доходность-Риск». Делается сравнение полученных результатов. Исследуются портфели, состоящие из акций различных компаний. В зависимости от отношения инвестора к риску можно выбрать соответствующие портфели.

Приводятся численные результаты эффективного распределения активов внутри портфеля для различных постановок оптимизационных задач.

Ключевые слова: портфель, риск, доходность, VaR, актив, оптимизация

Формул: 17; рис.: 4; табл.: 11; библ.: 14; приложений: 6

Introduction. The founder of financial instruments portfolio optimization is H.Markovitz [7]. J.Tobin [14], F.Black, W.Sharp [12]. Lintner [5], etc. were also engaged in the formation of various portfolios models.

In a risk management the approach on the basis of VaR indicator succeeded in H.Markovits portfolio formation problems solution. As a rule this indicator isn't used with reference to the markets which are in crisis [3].

Historically VaR concept was inseparably linked with delta-normal method of this indicator calculation. In its basis there is suggestion about the normal law of distribution of logarithmic type of market risk factors profitability. Using this method of VaR calculation the problem decision can be optimized.

Literature Review. The problem of constructing an optimal portfolio was considered by many scientists. Thus, Philippe Jorion [4] explores the risk and return relationship of active portfolios subject to a constraint on tracking-error volatility, which can also be interpreted in terms of value at risk. Such a constrained portfolio is the typical setup for active managers who are given the task of beating a benchmark. David Blanchett and Hal Ratner [2] introduce a new utility function to determine the optimal portfolio allocation for an income-focused investor which incorporates downside risk. Shalit H. and Yitzhaki S. [10] present the mean-Gini (MG) approach to analyze risky prospects and construct optimum portfolios. The proposed method has the simplicity of a mean-variance model and the main features of stochastic dominance efficiency.

In 2001 Basak S. and Shapiro A. [1] analyzed optimal, dynamic portfolio and wealth/consumption policies of utility maximizing investors who must also manage market-risk exposure using Value-at-Risk (VaR). They found that VaR risk managers often optimally choose a larger exposure to risky assets than non-risk managers and consequently incur larger losses when losses occur. Also they suggested an alternative risk-management model, based on the expectation of a loss, to remedy the shortcomings of VaR.

Rockafellar R. and Uryasev S. [8] proposed to use conditional value-at-risk (CVaR) as a measure of risk with significant advantages over value-at-risk (VaR). It provides optimization short-cuts which, through linear programming techniques, make practical many large-scale calculations that could otherwise be out of reach.

Methods. Let's consider possibility of VaR indicator application to the market risks evaluation and management. Let us have some portfolio consisting of open positions. Portfolio VaR for the given confidential level α and the holding period of positions *t* is defined as such a value which provides a possible losses coverage of the portfolio's holder for time t with probability $p = 1 - \alpha$.

One of the approaches of portfolio *VaR* calculation is based on the knowledge about individual *PVaR*:

$$VaR = \sqrt{PVaR^{T} \Omega PVaR} \tag{1}$$

where *PVaR* - a vector-column of position individual risks; Ω - correlation matrix.

Let's consider portfolios which can compose of instruments with fixed profitability and common stock. As a risk measure we will use VaR–indicator. In H.Markovitz classical portfolio the risk was estimated by the dispersion.

A. H. Markovitz Portfolio.

A1. Maximum efficiency portfolio.

It is necessary to find shares of the initial capital distribution $x_i \ (i = \overline{1,n})$ which maximize expected portfolio efficiency E_p, taking into account its separate components efficiencies $E_i = \ln(P_i^1 / P_i^0)$, where P_i^1 - is the stock price at the end of the period; P_i^0 - is the stock price at the beginning of the period. The required value of portfolio risk VaR^* is thus provided and the balance condition is satisfied :

$$\sum_{i=1}^{n} x_i = 1 \tag{2}$$

During the given portfolio formation regular risks and the investor's attitude to risk aren't considered. Portfolio includes only risky assets.

$$E_p = \sum_{i=1}^{n} x_i E_i \to \max$$
$$\begin{cases} \sqrt{PVaR^T \Omega PVaR} = VaR^* \\ \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{3}$$

A2. Maximum utility portfolio.

It is necessary to find shares of the initial capital distribution x_i which minimize portfolio risk (1). The required level of portfolio efficiency E^* is thus provided and the condition (2) is satisfied. During the given portfolio formation regular risks and the investor's attitude to risk aren't considered.

$$\sqrt{PVaR^T \Omega PVaR} \to \min$$
$$\begin{cases} \sum_{i=1}^{n} x_i E_i = E^* \\ \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{4}$$

A3. A maximum efficiency portfolio (taking into account risk-free assets).

The portfolio of A1 type is considered taking into account that in the portfolio there is risk-free financial instrument with a share x_0 and efficiency E_0. The theory of the best portfolio formation suggests that risk-free asset is the security which offers a fully predictable rate of return, calculated in monetary units, chosen for analysis, and the investor is not to review his decision within the given period. If there is no particular investor we can consider risk-free assets as such assets which offer investor a predictable rate of return within the period of trading. The portfolio (3) takes the form:

$$E_p = \sum_{i=0}^{n} x_i E_i \to \max$$
$$\begin{cases} \sqrt{PVaR^T \Omega PVaR} = VaR^* \\ \sum_{i=0}^{n} x_i = 1 \end{cases} \tag{5}$$

A4. The maximum utility portfolio taking into account investor's attitude to risk.

Determination of minimum risk portfolio taking into account investor's attitude to risk is reduced to the following optimization problem:

$$\sqrt{PVaR^T \Omega PVaR} \to \min$$
$$\begin{cases} 2\tau \sum_{i=1}^{n} x_i E_i = E^* \\ \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{6}$$

where $\tau = \frac{1}{R} \geq 0$ - reflects investor's tolerance to risk;
$R = -\omega U''(\omega)/U'(\omega)$ - is the relative risk measure of Arrow and Pratt;
$U(\omega)$ - is Neumann-Morgenstein utility function.

A5. Maximum utility portfolio in the process of investment strategy selection taking into account investor's liabilities.

Let's consider the one-period model, which characterizes investors activities in the financial market who form their assets portfolio taking into account their current and future obligations. These investors are, for example, pension funds and insurance companies that choose an investment strategy based on the ratio between their assets and liabilities.

Let's denote the initial value of the investor's net liabilities as L^0, and their value at the end of the considered time period as L^1. Then the liabilities increase rate, which in particular depends on such factors as risk-free assets rate of interest, inflation, economic growth indicator etc., will be represented by the following random variable: $R_L = \frac{L^1 - L^0}{L^0}$.

Let's suppose that the initial market value of investor's assets is equal A^0. Forming the investment portfolio which consists of n risky investments and has the profitability E^* investor increases his assets value to $A^1 = A^0(1+E^*)$ at the end of the considered period.

The difference between assets and liabilities at the initial moment of time is equal $S^0 = A^0 - L^0$, at the end of the period it is equal $S^1 = A^1 - L^1$. In accordance with the Markovitz approach, investment portfolio selection x_i taking into account current and future liabilities, is carried out so that it provides the required portfolio efficiency $E^s = \frac{S^1 - S^0}{S^0} = \frac{A^0}{A^0 - L^0} E^* - \frac{L^0}{A^0 - L^0} R_L$. Thus it is necessary to consider that the total sum of an investment portfolio, which is to be distributed, equals $M^s = A^1 - L^1$.

The portfolio in the given statement looks like as following:

$$\sqrt{PVaR^T \Omega PVaR} \to \min$$
$$\begin{cases} \sum_{i=1}^{n} x_i E_i = E^s \\ \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{7}$$

B. J.Tobin Portfolio

J.Tobin solved the Markovitz problem for the case when there are market risk-free securities included into the investment portfolio.

He suggested that if there are market risk-free securities, the solution of the optimal portfolio problem is simplified. Let's consider x_0 as the portion of capital invested in risk-free portfolio part, E_0 as risk-free rate of return (risk-free securities efficiency). Thus Markovitz problem of the efficient portfolio looks like as following

$$\sqrt{PVaR^T \Omega PVaR} \rightarrow \min$$

$$\begin{cases} x_0 E_0 + \sum_{i=1}^{n} x_i E_i = E^* \\ x_0 + \sum_{i=1}^{n} x_i = 1 \end{cases} \tag{8}$$

C. Black Portfolio

Black model considers a financial market where are no risk-free assets. Instead, the so-called zero beta coefficient portfolio is introduced.

This model allows formation of any portfolio in the case of constraints (2) existence. The feature of this model lies in possibility to obtain any, as much as big, efficiency, but at the expense of the rapidly growing risk.

Let's suppose that an investor, for the sake of future earnings, wants to increase his initial capital P^0 and invests the additional amount of capital P^*. Then, when buying different assets, he has $\sum_{i=1}^{n} P_i^0 = P^0 + P^*$ [13]. After formula conversion, we have $\sum_{i=1}^{n} x_i + x_{n+1} = 1$, where $x_{n+1} = -\frac{P^*}{P^0}$. Black portfolio looks like as following:

$$\sqrt{PVaR^T \Omega PVaR} \rightarrow \min$$

$$\sum_{i=1}^{n+1} x_i = 1 \tag{9}$$

To find a financial asset rate of return, taking into account market conditions, Black equation is used:

$$E_i = \rho^0 + \beta_i (E^m - \rho^0)$$

where $\beta_i = \frac{\text{cov}(E_i, E^m)}{\sigma^2(E^m)}$ - is called the beta of the *i*- asset;

ρ^0 - rate of return of the zero beta coefficient portfolio;

E^m - return on the market portfolio.

In practice, as the market portfolio rate of return financial indexes value is used (Dow Jones, Standard & Poor's 500, NYSE, NASDAQ, etc.) which provides generalized information about the financial market conditions.

Knowing the return on assets in the portfolio, we can form the corresponding model.

D. Tobin-Sharp-Lintner portfolio.

In the Sharp-Lintner model of financial assets pricing financial market is suggested to consist of the risk-free asset A^0 with efficiency E_0 and risk assets A_i with efficiencies E_i.

This model is more related to market structure rather than to the portfolio structure. It is believed that there is a risk-free asset (usually these are government securities or deposits in large banks) which efficiency does not depend on market conditions. If the return on risk-free asset is E_0, then its expected rate of return is also E_0, $\sigma^2(E_0) = 0$ and $\text{cov}(E_0, E_i) = 0$. All assets except zero asset A_0 are considered to be risky, i.e. $\sigma^2(E_i) > 0$. In this model portfolio with vector $\bar{x} = (x_0, x_1, \ldots x_n)$ if $x_0 \neq 1$ can be represented as a linear combination of risk-free and risky portfolio: $x = x_0 e_0 + (1 - x_0) y_0$, where $e_0 = (1,0,0,...0)$ is the risk-free portfolio that coincides with the risk-free asset; $y_0 = (0, \frac{x_1}{1-x_0},... \frac{x_n}{1-x_0})$ - is only risk including portfolio.

To find the profitability of risky assets, taking into account market changes, we use the following Sharp-Lintner equation:

$E_i = E_0 + \beta_i (E^m - E_0)$

where $\beta_i = \frac{\text{cov}(E_i, E^m)}{\sigma^2 (E^m)}$

The equation of Sharp-Lintner could be rewritten as following:

$E_i - E_0 = \beta_i (E^m - E_0)$

The difference ($E_i - E_0$) is called the risk award for the risky asset A_i, and ($E^m - E_0$) is called the risk award for the market portfolio.

In this formulation we have the following optimization problem:

$$\sqrt{PVaR^T \Omega PVaR} \to \min$$
$$\begin{cases} \sum_{i=0}^{n} x_i E_i = E^* \\ \sum_{i=0}^{n} x_i = 1 \end{cases} \tag{10}$$

E. Portfolios with additional linear restrictions.

The constructed models are basic in the formation of a real investment portfolio and reflect the impact of unsystematic risk. It is possible to supplement all received models with the following restrictions

E1. $-1 \le x_i \le 1$ - when short sales of a financial asset are permitted.

E2. $\sum x_i \le 1$ -instead of the balance condition (2).

E3. $\sum x_i = 0$ - instead of the balance condition (requiring the purchase of some assets by selling the other).

E4. Inequalities “ $\le$ “ can be used instead of " = " in the constraints including values of the efficiencies E^* or VaR^*.

E5. $x_i \ge 0$ - prohibits short sales of financial instruments.

E6. $\sum \beta_i x_i = \beta^m$ - takes into account the systematic risk

where β_i - is the regression ratio of the *i*- financial instrument on the market index;

β^m - is the required value of the total portfolio regression ratio on the market index.

Results and Discussion. Let's consider the stock market Dow30 (Appendix A). Let's assume that we plan to form an optimal portfolio of six companies' shares.

It is necessary to assess the market risk by *VaR* calculation both for the entire portfolio and for individual companies while investing for a specified number of days. For the analysis we use the historical prices of our portfolio shares in the Dow30 with the daily VaR calculation depth being one year period (01/09/2015- 01/09/2016).

Selection of the necessary financial instruments in the investment portfolio will be carried out with the help of factor analysis. Consider the formation of a portfolio of 6 shares based on the results of the factor analysis (Appendix B).

Portfolio based on Figure B1 (Portfolio A).

- Coca-Cola Company (KO)
- General Electric Company (GE)
- JPMorgan Chase & Co (JPM)
- Microsoft Corporation (MSFT)
- Pfizer Inc (PFE)
- Wal-Mart Stores Inc (WMT)

Portfolio based on Figure B2 (Portfolio B).

- Caterpillar Inc (CAT)
- Coca-Cola Company (KO)
- Goldman Sachs Group Inc (GS)
- Nike Inc (NKE)
- Pfizer Inc (PFE)
- UnitedHealth Group Incorporated (UNH)

The portfolio of the most profitable stocks (Portfolio C).

- 3M Company (MMM)
- EI du Pont de Nemours and Company (DD)
- General Electric Company (GE)
- Intel Corporation (INTC)
- Johnson & Johnson (JNJ)
- Microsoft Corporation (MSFT)

Let's suppose we have initial capital of K = 100 000 $ and its distribution is uniform between the issuers. Using correlation and covariance analysis, we find the correlation matrix of daily price changes.

With the use of appropriate mathematical tools, as well as taking into account historical data, we obtain the parameters of one-day 95% VaR estimates of the individual issuers as well as the whole portfolio (Appendix A) (Tables C1-C3).

Let's consider the decision of several kinds of optimization problems.

1. A problem about the maximum efficiency portfolio finding when short sales of the financial asset are prohibited (A1).

$$E_p = \sum_{i=1}^{n} x_i E_i \to \max$$
$$\begin{cases} \sqrt{PVaR^T \Omega PVaR} = VaR^* \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \tag{11}$$

As the limiting restriction of risk indicator we can use VaR^* of the portfolio at uniform distribution of stock (Tables C1-C3).

After solving (11) we obtain the maximum efficiency of the portfolios: portfolio A- 15,22%, portfolio B - 8,62%, portfolio C - 25,57% . In this case the optimal portfolio structure is represented in Appendix D (Tables D1-D3).

2. The problem of finding the maximum utility portfolio (A2).

$$\sqrt{PVaR^T \Omega PVaR} \to \min$$
$$\begin{cases} \sum_{i=1}^{n} x_i E_i = E^* \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \quad (12)$$

As the limiting restriction of profitability we can use E^* of the portfolio at uniform distribution of stock (Table C1-C3).

After solving (12) we obtain the minimum portfolios risk: portfolio A- 250,26$; portfolio B - 262,76$;, portfolio C - 277,53$. In this case the optimal portfolio structure is represented in Appendix E (Tables E1-E3).

We construct an optimal curve " Profitability - Risk" for three portfolios.

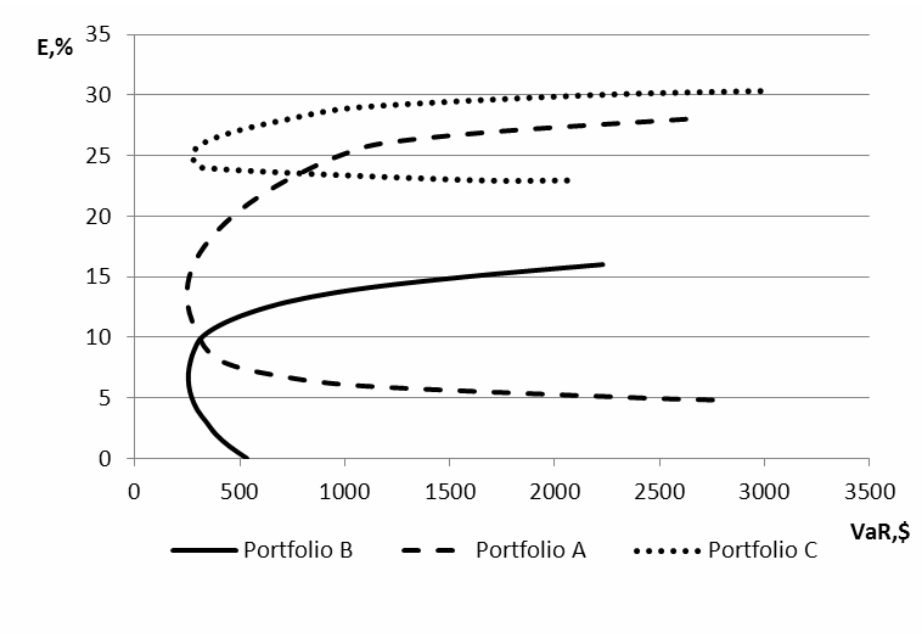

Figure1. Optimal curves " Profitability - Risk"

Consider the risk indicator in a form:

$$VaR_{\%} = \frac{VaR}{P} = k_{1-\alpha}\sigma \quad (13)$$

where P – the share price at the opening, $; $k_{1-\alpha}$ – quantile of the distribution of returns with a confidence level α.

For example, for standart normal distribution: $k_{1-0.05} = 1{,}645$; $k_{1-0.01} = 2{,}326$.

Let's consider the most useful portfolio:

$$\sqrt{PVaR_{\%}^{\ T}\Omega PVaR_{\%}} \to \min$$

$$\begin{cases} \sum_{i=1}^{n} x_i E_i = E^* \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \end{cases} \tag{14}$$

As an example, we find the optimal solution (Appendix F) for portfolio B and compare it with the previously obtained (Table E2).

For the portfolio B we obtained the next regression:

$$VaR_{\%} = 163{,}9128 + 0{,}6568VaR \text{ , } (R^2 = 0{,}996 \text{ ; } F = 3554{,}5) \tag{15}$$

From these results it can be concluded that the formation of equal profitability of the portfolios of the same set of companies the optimal proportion of shares in the portfolio varies according to the risk criterion.

When considering the time slot *t* the risk of portfolio can be founded using the formula:

$$VaR_t = VaR\sqrt{t} \tag{16}$$

We assume that the portfolio is efficient if the profitability of the portfolio over its relative magnitude of risk $RVaR = \frac{VaR_t}{K}$:

$$E \geq RVaR \tag{17}$$

Figure 2 shows the distribution of efficient portfolios on the time interval t = 254 if the condition (17) is true.

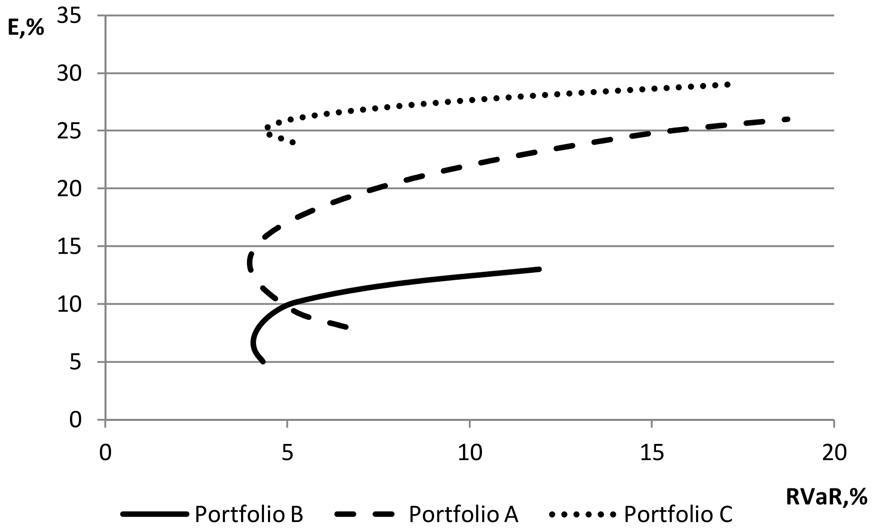

Figure 2. Efficient portfolios

Analysis of the data allows to obtain an optimal portfolio for a given level of return and risk. In the formation of the portfolio must also take into account the investor's attitude to risk, based on studies of the utility function ([9]).

Conclusion. In this paper, various optimization problems statements on the basis of H.Markovits portfolio theory are provided. The maximum efficiency and the maximum utility portfolios are considered, taking into account the investor's attitude to risk. Linear restrictions, reflecting the systematic and unsystematic risk, as well as the short sales prohibition are provided. Numerical results of the portfolio optimization are given, with risk assessment being based on the index.

Appendix A

Table A1

Companies and their profitability in the Dow 30.

№	Company	Profitability,% (01/09/2015-01/09/2016)	Opening prices, 02/09/2016, $
1	3M Company (MMM)	23,64	180,53
2	American Express Company (AXP)	-16,73	65,06
3	Apple Inc (AAPL)	-5,39	107,7
4	Boeing Company (BA)	-0,57	130,89
5	Caterpillar Inc (CAT)	6,37	82,19
6	Chevron Corporation (CVX)	21,35	100,89
7	Cisco Systems Inc (CSCO)	19,95	31,65
8	Coca-Cola Company (KO)	9,77	43,52
9	EI du Pont de Nemours and Company (DD)	30,35	70,18
10	Exxon Mobil Corporation (XOM)	14,43	87,42
11	General Electric Company (GE)	22,95	31,25
12	Goldman Sachs Group Inc (GS)	-11,21	168,52
13	Home Depot Inc (HD)	14,24	134,77
14	International Business Machines (IBM)	7,65	159,88
15	Intel Corporation (INTC)	23,31	36,21
16	Johnson & Johnson (JNJ)	23,69	119,36
17	JPMorgan Chase & Co (JPM)	4,83	67,4
18	McDonald’s Corporation (MCD)	19,45	115,93
19	Merck & Company Inc (MRK)	15,57	62,59
20	Microsoft Corporation (MSFT)	28,09	57,67
21	Nike Inc (NKE)	4,69	58,63
22	Pfizer Inc (PFE)	7,39	34,78
23	Procter & Gamble Company (PG)	22,32	88,48
24	The Travelers Companies Inc (TRV)	17,32	118,06
25	United Technologies Corporation (UTX)	15,29	107,19
26	UnitedHealth Group Incorporated (UNH)	16,01	136,5
27	Verizon Communications Inc (VZ)	13,34	52,8
28	Visa Inc (V)	13,18	81,57
29	Wal-Mart Stores Inc (WMT)	11,81	72,99
30	Walt Disney Company (DIS)	-7,75	94,75

Appendix B

The results of the application of factor analysis for the Dow 30 stocks.

Factor Loadings (Varimax raw) (Spreadsheet2)
Extraction: Principal components
(Marked loadings are > ,700000)

Variable	Factor 1	Factor 2	Factor 3
Var1	0,635609	-0,209635	0,709063
Var2	-0,420328	0,855442	0,191558
Var3	-0,137099	0,854112	-0,036342
Var4	0,158267	0,931103	0,082510
Var5	0,275863	0,072408	0,875481
Var6	0,667318	-0,137294	0,659311
Var7	0,425051	0,220821	0,809186
Var8	0,802700	-0,388056	0,235110
Var9	0,858546	0,119157	0,356854
Var10	0,557832	-0,286974	0,667449
Var11	0,932939	-0,051492	0,280382
Var12	-0,236928	0,944418	-0,109262
Var13	0,814897	0,049713	0,427775
Var14	0,123843	0,058354	0,936254
Var15	0,593458	0,555943	0,360094
Var16	0,601633	-0,338097	0,698249
Var17	0,272066	0,752178	0,455541
Var18	0,823099	-0,440355	0,050090
Var19	0,423442	0,036781	0,828514
Var20	0,888268	0,097655	0,110638
Var21	0,228818	0,637789	-0,583454
Var22	0,146630	0,285880	0,825581
Var23	0,732753	-0,467471	0,413632
Var24	0,862353	0,049593	0,418416
Var25	0,549616	0,092647	0,783593
Var26	0,334445	-0,300986	0,865833
Var27	0,500450	-0,616970	0,532128
Var28	0,703776	0,382799	0,454502
Var29	0,174801	-0,619834	0,711588
Var30	0,067484	0,886828	-0,195910
Expl.Var	9,608328	7,293107	9,474001
Prp.Totl	0,320278	0,243104	0,315800

Figure B1. Factor in the price based on load

Factor Loadings (Varimax raw) (Spreadsheet2)
Extraction: Principal components
(Marked loadings are > ,700000)

Variable	Factor 1	Factor 2	Factor 3	Factor 4
Var1	0,518905	0,181189	0,521510	0,200814
Var2	0,488899	0,367313	-0,179790	0,315628
Var3	0,423832	0,271890	0,136260	0,355363
Var4	0,591756	0,196689	0,253160	0,333920
Var5	0,830900	0,079671	0,095377	0,086439
Var6	0,691110	0,135139	0,452310	-0,114331
Var7	0,503745	0,143329	0,354084	0,377634
Var8	0,229357	0,096320	0,712455	0,214133
Var9	0,621906	0,034280	0,132754	0,150157
Var10	0,662037	0,138845	0,495776	-0,146501
Var11	0,605357	0,291519	0,328644	0,335369
Var12	0,701059	0,363119	0,094476	0,348049
Var13	0,204151	0,256257	0,317057	0,682318
Var14	0,594568	0,131609	0,281728	0,267899
Var15	0,614994	0,229965	0,251309	0,374400
Var16	0,297393	0,521341	0,563208	0,052317
Var17	0,687291	0,360416	0,191008	0,337836
Var18	0,105328	0,165220	0,610324	0,420805
Var19	0,304507	0,668010	0,259087	0,134025
Var20	0,404115	0,321820	0,274066	0,445553
Var21	0,135397	0,156737	0,132650	0,719928
Var22	0,123550	0,817092	0,171549	0,113715
Var23	0,268955	0,245496	0,667535	0,238728
Var24	0,439600	0,328511	0,464210	0,317773
Var25	0,594131	0,145376	0,267250	0,353781
Var26	0,156078	0,747533	0,050552	0,278116
Var27	0,300247	0,078734	0,668568	0,162817
Var28	0,377287	0,433705	0,196705	0,554636
Var29	-0,025553	0,119950	0,461789	0,312242
Var30	0,490113	0,215951	0,311745	0,405754
Expl.Var	6,946724	3,382407	4,303634	3,540255
Prp.Totl	0,231557	0,112747	0,143454	0,118009

Figure B2. Factor loadings on the basis of profitability

Appendix C

Estimates of VaR 99% level of the portfolio at uniform distribution of stock.

Table C1

Portfolio A.

Indicators	Company						Portfolio
	KO	GE	JPM	MSFT	PFE	WMT	
Profitability,%	9,77	22,95	4,83	28,09	7,39	11,81	14,14
VaR, $	42,51	57,93	76,46	75,06	59,10	65,20	264,71
Stock shares	1/6	1/6	1/6	1/6	1/6	1/6	1
Number of stocks	383	533	247	289	479	228	

Table C2

Portfolio B.

Indicators	Company						Portfolio
	CAT	KO	GS	NKE	PFE	UNH	
Profitability,%	6,37	9,77	-11,21	4,69	7,39	16,01	5,51
VaR, $	83,77	42,51	80,33	73,23	59,10	61,99	275,36
Stock shares	1/6	1/6	1/6	1/6	1/6	1/6	1
Number of stocks	203	383	99	284	479	122	

Table C3

Portfolio C.

Indicators	Company						Portfolio
	MMM	DD	GE	INTC	JNJ	MSFT	
Profitability,%	23,64	30,35	22,95	23,31	23,69	28,09	25,34
VaR, $	50,62	83,81	57,93	67,71	41,50	75,06	287,19
Stock shares	1/6	1/6	1/6	1/6	1/6	1/6	1
Number of stocks	92	237	533	460	140	289	

Appendix D

Optimal stock allocation in the maximum efficiency portfolio.

Table D1

Portfolio A.

Indicators	Company						Portfolio
	KO	GE	JPM	MSFT	PFE	WMT	
Profitability,%	9,77	22,95	4,83	28,09	7,39	11,81	15,22
VaR, $	56,69	71,07	24,23	100,04	58,81	68,17	264,71
Stock shares	0,1925	0,1846	0,0938	0,1924	0,1663	0,1704	1
Number of stocks	442	591	139	334	478	233	

Table D2

Portfolio B.

Indicators	Company						Portfolio
	CAT	KO	GS	NKE	PFE	UNH	
Profitability,%	6,37	9,77	-11,21	4,69	7,39	16,01	8,62
VaR, $	64,83	113,43	4,10	59,35	69,47	100,98	275,36
Stock shares	0,1466	0,2723	0,0377	0,1500	0,1807	0,2127	1
Number of stocks	178	626	22	256	520	156	

Table D3

Portfolio C.

Indicators	Company						Portfolio
	MMM	DD	GE	INTC	JNJ	MSFT	
Profitability,%	23,64	30,35	22,95	23,31	23,69	28,09	25,57
VaR, $	48,71	107,35	35,42	35,86	73,19	82,70	287,19
Stock shares	0,1635	0,1886	0,1303	0,1213	0,2213	0,1749	1
Number of stocks	91	269	417	335	185	303	

Appendix E

Optimal stock allocation in the maximum utility portfolio.

Table E1

Portfolio A.

Indicators	Company						Portfolio
	KO	GE	JPM	MSFT	PFE	WMT	
Profitability,%	9,77	22,95	4,83	28,09	7,39	11,81	14,14
VaR, $	75,69	59,68	33,81	59,98	61,58	74,79	250,26
Stock shares	0,2224	0,1692	0,1108	0,1490	0,1701	0,1785	1
Number of stocks	511	541	164	258	489	245	

Table E2

Portfolio B.

Indicators	Company						Portfolio
	CAT	KO	GS	NKE	PFE	UNH	
Profitability,%	6,37	9,77	-11,21	4,69	7,39	16,01	5,51
VaR, $	54,26	83,14	76,29	66,85	67,03	39,85	262,76
Stock shares	0,1341	0,2331	0,1624	0,1592	0,1775	0,1336	1
Number of stocks	163	536	96	272	510	98	

Table E3

Portfolio C.

Indicators	Company						Portfolio
	MMM	DD	GE	INTC	JNJ	MSFT	
Profitability,%	23,64	30,35	22,95	23,31	23,69	28,09	25,34
VaR, $	53,87	84,74	42,81	41,73	77,43	68,06	277,53
Stock shares	0,1719	0,1676	0,1433	0,1308	0,2277	0,1587	1
Number of stocks	95	239	458	361	191	275	

Appendix F

Optimal stock allocation in the maximum utility portfolio.

Таблица F1

Portfolio B.

Indicators	Company						Portfolio
	CAT	KO	GS	NKE	PFE	UNH	
Profitability,%	6,37	9,77	-11,21	4,69	7,39	16,01	5,51
$VaR_{\%}$,%	71,27	86,86	85,41	65,77	44,57	122,80	326,35
Stock shares	0,1394	0,1572	0,2231	0,1209	0,0854	0,2741	1
Number of stocks	170	361	132	206	245	201	

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